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Analysis of a Control System using Hurwitz Stability Criterion

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Abstract
The Hurwitz Stability Criterion is a method employed in analysis of control system. It is used in determining whether all the roots of the characteristic equation of a continuous system have negative real parts. It is a method that only shows interest in the determinants values of a characteristic equation of a continuous system. The mathematical modelling of milling machine in 3-degree of freedom (DOF) using its dynamic parameters presented in this work. The design method employed in this work utilized various software to design, model and analyzed the milling machine. Linear time-invariant (LTI) ordinary differential equations (ODEs) with forcing using both time-domain and Laplace transform methods were among the methods used to develop the mathematical model. With this model, analysis was also carried out using linear control system analysis/criterion via Hurwitz stability criterion. The analysis of the model using the linear control system stability criterion showed that the mathematical model of the milling machine in 3-DOF using its dynamic parameters was stable for the stability criterion confirming the accuracy of the model. It thus, serve as a guide for engineers, technicians and machinists during milling operation and also contributes to the body of knowledge in chatter vibration in milling machine. In this paper, the determinants formed from the coefficients of a 6th-order characteristic equation using the Hurwitz stability criterion was presented. Its stability was established only when all the roots of the characteristic equation have negative real parts and only if all the determinants are greater than zero.

Keywords: *Characteristic equation, Negative real parts, Determinants, Hurwitz stability criterion*

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1. Introduction

The Hurwitz polynomial (named after Adolf Hurwitz) is a polynomial whose zeros i.e. (roots) are located in the left half-plane of the complex plane or on the imaginary axis, that is, the real part of every root is zero or negative. Such a polynomial must have coefficients that are positive real numbers. The term is sometimes restricted to polynomials whose roots have real parts that are strictly negative, excluding the axis (i.e., a Hurwitz stable polynomial). Hurwitz polynomials are important in control systems theory because they represent the characteristic equations of stable linear systems. It can be determined whether a polynomial is Hurwitz by solving the equation to find the roots, or from the coefficients without solving the equation by the Routh–Hurwitz stability criterion.

For a polynomial to be Hurwitz, it is necessary but not sufficient that all of its coefficients be positive (except for second-degree polynomials which also do not imply sufficiency). A necessary and sufficient condition that a polynomial is Hurwitz is that it passes the Routh–Hurwitz stability criterion (Ogata 2010). A given polynomial can be efficiently tested to be Hurwitz or not by using the

Routh continued fraction expansion technique. The Hurwitz criterion is applied using the determinants formed from the coefficients of the characteristic equation. It is assumed that the first coefficient, a_n, is positive (Distefano et al., 2010).

Saleh (2013) presented a method using the energy balance with 3-DOF mode in two orthogonal x and y direction. In this research, a new approach based on control system analysis using Hurwitz stability criterion has been carried out. A milling model has been developed to consider variable dimension of the dynamic parameter of 3-DOF modes in three orthogonal, x, y and z direction. This technique was not considered before.

2. Materials and methods

The control system analysis was carried out in this research work. To achieve this, the method of Hurwitz stability criterion was considered.

(a) Hurwitz Stability Criterion:

Consider the characteristic equation given by:

$$a_0 s^n + a_1 s^{n-1} + \dots\dots + a_{n-1} s + a_n = 0 \qquad (1)$$

The Hurwitz stability criterion, gives conditions for all the roots to have negative real parts in terms of the coefficients of the polynomial. It states that for all the roots to have negative real parts, all the coefficients a's must be positive. This is a necessary condition but not a sufficient condition. If this condition is not satisfied, it indicates that some of the roots have positive real parts or are imaginary or zero. The necessary and sufficient condition is that all the successive principal minors of the determinant, Δ_n be positive. If all the determinants are positive, and a_0 >0 as already assumed, then the equilibrium state of the system whose characteristic equation is given in Equation (1) is asymptotically stable. Hurwitz criterion states that all the roots of the characteristic equation have negative real parts if and only if $\Delta_i > 0, i = 1, 2, \dots., n$.

In this paper, a 6th-order characteristic equation is considered. Its stability is determined whether all the roots of the characteristic equation have negative real part.

(b) Stability Analysis for Hurwitz Stability Criterion:

Considering the stability of the characteristic equation of the 6th-order polynomial below.

$$0.027s^6 + 87.3s^5 + 25.68 \times 10^6 s^4 + 54.932 \times 10^9 s^3 + 6.832 \times 10^{15} s^2 + 4.632 \times 10^{18} s + 0.444 \times 10^{24} = 0$$

Since all the coefficients of the polynomial are positive, the coefficients in the determinant are arranged as follow:

$$\Delta_{nx} = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 & 0 \\ a_0 & a_2 & a_4 & a_6 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 & 0 \\ 0 & a_0 & a_2 & a_4 & a_6 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 0 & a_0 & a_2 & a_4 & a_6 \end{vmatrix} \qquad (2)$$

where the a_n's have the following values.

$a_0 = 0.027$, $a_1 = 87.3$, $a_2 = 25.68 \times 10^6$, $a_3 = 54.932 \times 10^9$, $a_4 = 6.832 \times 10^{15}$, $a_5 = 4.632 \times 10^{18}$, $a_6 = 0.444 \times 10^{24}$

By substituting the value of the Routh's array coefficients in Equation (2), the following determinant were obtained.

$\Delta_1 = |a_1| = 87.3$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} = a_1 a_2 - a_0 a_3$$

$= |(87.3 \times 25.680 \times 10^6) - (0.027 \times 54.932 \times 10^9)|$

$= 0.7587 \times 10^9$

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_1 \end{vmatrix}$$

$= a_1|a_2 a_3 - a_1 a_4| - a_3|a_0 a_3| + a_5|a_0 a_1|$

$=87.3|(25.680 \times 10^6 \times 54.932 \times 10^9) - (87.3 \times 6.832 \times 10^{15})| - 54.932 \times 10^9|0.027 \times 54.932 \times 10^9| + 4.632 \times 10^{18}|0.027 \times 87.3|$

$= 0.5251 \times 10^{18}$

$$\Delta_4 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} a_2 & a_4 & a_6 \\ a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \end{vmatrix} - a_3 \begin{vmatrix} a_0 & a_4 & a_6 \\ 0 & a_3 & a_5 \\ 0 & a_2 & a_4 \end{vmatrix} + a_5 \begin{vmatrix} a_0 & a_2 & a_6 \\ 0 & a_1 & a_5 \\ 0 & a_0 & a_4 \end{vmatrix}$$

$$= a_1[a_2(a_3a_4 - a_2a_5) - a_4(a_1a_4 - a_0a_5) + a_6(a_1a_2 - a_0a_3)] - a_3[a_0(a_3a_4 - a_2a_5)] + a_5[a_0(a_1a_4 - a_0a_5)]$$

$$= 87.3\big[25.680 \times 10^6\big|(54.932 \times 10^9 \times 6.832 \times 10^{15}) - (25.680 \times 10^6 \times 4.632 \times 10^{18})\big| - 6.832 \times 10^{15}\big|(87.3 \times 6.832 \times 10^{15}) - (0.027 \times 4.632 \times 10^{18})\big| + 0.444 \times 10^{24}|(87.3 \times 25.680 \times 10^6) - (0.027 \times 54.932 \times 10^9|\big] - 54.932 \times 10^9\big[0.027\big|(54.932 \times 10^9 \times 6.832 \times 10^{15}) - (25.680 \times 10^6 \times 4.632 \times 10^{18})\big|\big] + 4.632 \times 10^{18}\big[0.027\big|(87.3 \times 6.832 \times 10^{15}) - (0.027 \times 4.632 \times 10^{18})\big|\big]$$

$$= 1.711 \times 10^{33}$$

$$\Delta_5 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ a_0 & a_2 & a_4 & a_6 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & a_0 & a_2 & a_4 & a_6 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} a_2 & a_4 & a_6 & 0 \\ a_1 & a_3 & a_5 & 0 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \end{vmatrix} - a_3 \begin{vmatrix} a_0 & a_4 & a_6 & 0 \\ 0 & a_3 & a_5 & 0 \\ 0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \end{vmatrix} + a_5 \begin{vmatrix} a_0 & a_2 & a_6 & 0 \\ 0 & a_1 & a_5 & 0 \\ 0 & a_0 & a_4 & a_6 \\ 0 & 0 & a_3 & a_5 \end{vmatrix}$$

$$= a_1 \left[a_2 \begin{vmatrix} a_3 & a_5 & 0 \\ a_2 & a_4 & a_6 \\ a_1 & a_3 & a_5 \end{vmatrix} - a_4 \begin{vmatrix} a_1 & a_5 & 0 \\ a_0 & a_4 & a_6 \\ 0 & a_3 & a_5 \end{vmatrix} + a_6 \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & a_6 \\ 0 & a_1 & a_5 \end{vmatrix} \right] - a_3 \left[a_0 \begin{vmatrix} a_3 & a_5 & 0 \\ a_2 & a_4 & a_6 \\ a_1 & a_3 & a_5 \end{vmatrix} - a_4 \begin{vmatrix} 0 & a_5 & 0 \\ 0 & a_4 & a_6 \\ 0 & a_3 & a_5 \end{vmatrix} + a_6 \begin{vmatrix} 0 & a_3 & 0 \\ 0 & a_2 & a_6 \\ 0 & a_1 & a_5 \end{vmatrix} \right] + a_5 \left[a_0 \begin{vmatrix} a_1 & a_5 & 0 \\ a_0 & a_4 & a_6 \\ 0 & a_3 & a_5 \end{vmatrix} - a_2 \begin{vmatrix} 0 & a_5 & 0 \\ 0 & a_4 & a_6 \\ 0 & a_3 & a_5 \end{vmatrix} + a_6 \begin{vmatrix} 0 & a_1 & 0 \\ 0 & a_0 & a_6 \\ 0 & 0 & a_5 \end{vmatrix} \right]$$

$$= [a_1(a_2|a_3(a_4a_5 - a_3a_6) - a_5(a_2a_5 - a_1a_6))| - a_4|a_1(a_4a_5 - a_3a_6) - a_5(a_0a_5)| + a_6|a_1(a_2a_5 - a_1a_6) - a_3(a_0a_5)|] - [a_3(a_0|a_3(a_4a_5 - a_3a_6) - a_5(a_2a_5 - a_1a_6)|] + [a_5(a_0|a_1(a_4a_5 - a_3a_6) - a_5(a_0a_5))|]$$

$$= \big[87.3\,(25.680 \times 10^6\big|54.932 \times 10^9(6.832 \times 10^{15} \times 4.632 \times 10^{18} - 54.932 \times 10^9 \times 0.444 \times 10^{24}) - 4.632 \times 10^{18}(25.680 \times 10^6 \times 4.632 \times 10^{18} - 87.3 \times 0.444 \times 10^{24}))\big| - 6.832 \times 10^{15}\big|87.3\,(6.832 \times 10^{15} \times 4.632 \times 10^{18} - 54.932 \times 10^9 \times 0.444 \times 10^{24}) - 4.632 \times 10^{18}(0.027 \times 4.632 \times 10^{18})\big| + 0.444 \times 10^{24}|87.3\,(25.680 \times 10^6 \times 4.632 \times 10^{18} - 87.3 \times 0.444 \times 10^{24}) - 54.932 \times 10^9(0.027 \times 4.632 \times 10^{18})|\big] - \big[54.932 \times 10^9(0.027\big|54.932 \times 10^9(6.832 \times 10^{15} \times 4.632 \times 10^{18} - 54.932 \times 10^9 \times 0.444 \times 10^{24}) - 4.632 \times 10^{18}(25.680 \times 10^6 \times 4.632 \times 10^{18} - 87.3 \times 0.444 \times 10^{24})\big|\big] + \big[4.632 \times 10^{18}(0.027\big|87.3\,(6.832 \times 10^{15} \times 4.632 \times 10^{18} - 54.932 \times 10^9 \times 0.444 \times 10^{24}) - 4.632 \times 10^{18}(0.027 \times 4.632 \times 10^{18}))\big|\big]$$

$$= 1.427 \times 10^{50}$$

$$\Delta_6 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 & 0 \\ a_0 & a_2 & a_4 & a_6 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 & 0 \\ 0 & a_0 & a_2 & a_4 & a_6 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 0 & a_0 & a_2 & a_4 & a_6 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} a_2 & a_4 & a_6 & 0 & 0 \\ a_1 & a_3 & a_5 & 0 & 0 \\ a_0 & a_2 & a_4 & a_6 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & a_0 & a_2 & a_4 & a_6 \end{vmatrix} -$$

$$a_3 \begin{vmatrix} a_0 & a_4 & a_6 & 0 & 0 \\ 0 & a_3 & a_5 & 0 & 0 \\ 0 & a_2 & a_4 & a_6 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & a_0 & a_2 & a_4 & a_6 \end{vmatrix} +$$

$$a_5 \begin{vmatrix} a_0 & a_2 & a_6 & 0 & 0 \\ 0 & a_1 & a_5 & 0 & 0 \\ 0 & a_0 & a_4 & a_6 & 0 \\ 0 & 0 & a_3 & a_5 & 0 \\ 0 & 0 & a_2 & a_4 & a_6 \end{vmatrix}$$

$$= a_1 \left[a_2 \begin{vmatrix} a_3 & a_5 & 0 & 0 \\ a_2 & a_4 & a_6 & 0 \\ a_1 & a_3 & a_5 & 0 \\ a_0 & a_2 & a_4 & a_6 \end{vmatrix} - \right.$$

$$a_4 \begin{vmatrix} a_1 & a_5 & 0 & 0 \\ a_0 & a_4 & a_6 & 0 \\ 0 & a_3 & a_5 & 0 \\ 0 & a_2 & a_4 & a_6 \end{vmatrix} +$$

$$\left. a_6 \begin{vmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_6 & 0 \\ 0 & a_1 & a_5 & 0 \\ 0 & a_0 & a_4 & a_6 \end{vmatrix} \right] -$$

$$a_3 \left[a_0 \begin{vmatrix} a_3 & a_5 & 0 & 0 \\ a_2 & a_4 & a_6 & 0 \\ a_1 & a_3 & a_5 & 0 \\ a_0 & a_2 & a_4 & a_6 \end{vmatrix} - \right.$$

$$a_4 \begin{vmatrix} 0 & a_5 & 0 & 0 \\ 0 & a_4 & a_6 & 0 \\ 0 & a_3 & a_5 & 0 \\ 0 & a_2 & a_4 & a_6 \end{vmatrix} +$$

$$\left. a_6 \begin{vmatrix} 0 & a_3 & 0 & 0 \\ 0 & a_2 & a_6 & 0 \\ 0 & a_1 & a_5 & 0 \\ 0 & a_0 & a_4 & a_6 \end{vmatrix} \right] +$$

$$a_5 \left[a_0 \begin{vmatrix} a_1 & a_5 & 0 & 0 \\ a_0 & a_4 & a_6 & 0 \\ 0 & a_3 & a_5 & 0 \\ 0 & a_2 & a_4 & a_6 \end{vmatrix} - a_2 \begin{vmatrix} 0 & a_5 & 0 & 0 \\ 0 & a_4 & a_6 & 0 \\ 0 & a_3 & a_5 & 0 \\ 0 & a_2 & a_4 & a_6 \end{vmatrix} \right.$$

$$\left. + \; a_6 \begin{vmatrix} 0 & a_1 & 0 & 0 \\ 0 & a_0 & a_6 & 0 \\ 0 & 0 & a_5 & 0 \\ 0 & 0 & a_4 & a_6 \end{vmatrix} \right]$$

$$= a_1 \left[a_2 \left(a_3 \begin{vmatrix} a_4 & a_6 & 0 \\ a_3 & a_5 & 0 \\ a_2 & a_4 & a_6 \end{vmatrix} - a_5 \begin{vmatrix} a_2 & a_6 & 0 \\ a_1 & a_5 & 0 \\ a_0 & a_4 & a_6 \end{vmatrix} \right) - \right.$$

$$a_4 \left(a_1 \begin{vmatrix} a_4 & a_6 & 0 \\ a_3 & a_5 & 0 \\ a_2 & a_4 & a_6 \end{vmatrix} - \right.$$

$$\left. a_5 \begin{vmatrix} a_0 & a_6 & 0 \\ 0 & a_5 & 0 \\ 0 & a_4 & a_6 \end{vmatrix} \right) +$$

$$a_6 \left(a_1 \begin{vmatrix} a_2 & a_6 & 0 \\ a_1 & a_5 & 0 \\ a_0 & a_4 & a_6 \end{vmatrix} - \right.$$

$$\left. \left. a_3 \begin{vmatrix} a_0 & a_6 & 0 \\ 0 & a_5 & 0 \\ 0 & a_4 & a_6 \end{vmatrix} \right) \right] -$$

$$a_3 \left[a_0 \left(a_3 \begin{vmatrix} a_4 & a_6 & 0 \\ a_3 & a_5 & 0 \\ a_2 & a_4 & a_6 \end{vmatrix} - \right. \right.$$

$$\left. a_5 \begin{vmatrix} a_2 & a_6 & 0 \\ a_1 & a_5 & 0 \\ a_0 & a_4 & a_6 \end{vmatrix} \right) -$$

$$a_4 \left(- a_5 \begin{vmatrix} 0 & a_6 & 0 \\ 0 & a_5 & 0 \\ 0 & a_4 & a_6 \end{vmatrix} \right) +$$

$$\left. a_6 \left(- a_3 \begin{vmatrix} 0 & a_6 & 0 \\ 0 & a_5 & 0 \\ 0 & a_4 & a_6 \end{vmatrix} \right) \right] +$$

$$a_5 \left[a_0 \left(a_1 \begin{vmatrix} a_4 & a_6 & 0 \\ a_3 & a_5 & 0 \\ a_2 & a_4 & a_6 \end{vmatrix} - \right. \right.$$

$$\left. a_5 \begin{vmatrix} a_0 & a_6 & 0 \\ 0 & a_5 & 0 \\ 0 & a_4 & a_6 \end{vmatrix} \right) -$$

$$a_2\left(-a_5\begin{vmatrix}0 & a_6 & 0\\0 & a_5 & 0\\0 & a_4 & a_6\end{vmatrix}\right)+$$

$$a_6\left(-a_1\begin{vmatrix}0 & a_6 & 0\\0 & a_5 & 0\\0 & a_4 & a_6\end{vmatrix}\right)\Bigg]$$

$= a_1[a_2|a_3\{a_4(a_5a_6) - a_6(a_3a_6)\} -$

$a_5\{a_2(a_5a_6) - a_6(a_1a_6)\}| -$

$a_4|a_1\{a_4(a_5a_6) - a_6(a_3a_6)\} -$

$a_5\{a_0(a_5a_6)\}| + a_6|a_1\{a_2(a_5a_6) -$

$a_6(a_1a_6)\} - a_3\{a_0(a_5a_6\}|] -$

$a_3[a_0|a_3\{a_4(a_5a_6) - a_6(a_3a_6)\} -$

$a_5\{a_2(a_5a_6) - a_6(a_1a_6)\}|] +$

$a_5[a_0|a_1\{a_4(a_5a_6) - a_6(a_3a_6)\} -$

$a_5\{a_0(a_5a_6)\}|]$

$= 87.3\big[25.68 \times 10^6\big|54.932 \times 10^9\{6.832 \times$

$10^{15}(4.632 \times 10^{18} \times 0.444 \times 10^{24}) -$

$0.444 \times 10^{24}(54.932 \times 10^9 \times 0.444 \times 10^{24})\} -$

$4.632 \times 10^{18}\{25.68 \times 10^6(4.632 \times 10^{18} \times$

$0.444 \times 10^{24}) - 0.444 \times 10^{24}(87.3 \times 0.444 \times$

$10^{24})\}\big| - 6.832 \times 10^{15}\big|87.3\{6.832 \times$

$10^{15}(4.632 \times 10^{18} \times 0.444 \times 10^{24}) -$

$0.444 \times 10^{24}(54.932 \times 10^9 \times 0.444 \times$

$10^{24})\} - 4.632 \times 10^{18}\{0.027(4.632 \times 10^{18} \times$

$0.444 \times 10^{24})\}\big| + 0.444 \times 10^{24}|87.3\{25.68 \times$

$10^6(4.632 \times 10^{18} \times 0.444 \times 10^{24}) -$

$0.444 \times 10^{24}(87.3 \times 0.444 \times 10^{24})\} -$

$54.932 \times 10^9\{0.027(4.632 \times 10^{18} \times 0.444 \times$

$10^{24}\}|\big] - 54.932 \times 10^9\big[0.027\big|54.932 \times$

$10^9\{6.832 \times 10^{15}(4.632 \times 10^{18} \times 0.444 \times$

$10^{24}) - 0.444 \times 10^{24}(54.932 \times 10^9 \times 0.444 \times$

$10^{24})\} - 4.632 \times 10^{18}\{25.68 \times 10^6(4.632 \times$

$10^{18} \times 0.444 \times 10^{24}) - 0.444 \times 10^{24}(87.3 \times$

$0.444 \times 10^{24})\}\big|\big] + 4.632 \times$

$10^{18}\big[0.027\big|87.3\{6.832 \times 10^{15}(4.632 \times 10^{18} \times$

$0.444 \times 10^{24}) - 0.444 \times 10^{24}(54.932 \times 10^9 \times$

$0.444 \times 10^{24})\} - 4.632 \times 10^{18}\{0.027(4.632 \times$

$10^{18} \times 0.444 \times 10^{24})\}\big|\big]$

$= 5.45 \times 10^{73}$

3. Results and discussion

Hurwitz stability criterion is only interested in the determinant values to ascertain the stability of a system. Hence, the determinant values are:

$\Delta_1 = 87.3$, $\Delta_2 = 0.7587\times 10^9$, $\Delta_3 = 0.5251\times 10^{18}$, $\Delta_4 = 1.711\times 10^{33}$, $\Delta_5 = 1.427\times 10^{50}$, $\Delta_6 = 5.45\times 10^{73}$

The result presented above showed the determinant of the 6th-order characteristic equation using the Hurwitz stability criterion. Since all the determinant values, $\Delta_i's$ have negative real parts i.e., $\Delta_i's > 0$. It therefore shows that the system is stable confirming the accuracy of the model. Literatures on control system analysis is very limited. Thus, this study is unique in itself. However, Saleh (2013) related study presented the energy balance with 3-DOF mode in two orthogonal x and y direction. An energy balance approach as an alternative method of exploring the system stability from time domain simulation data. In this research, a new approach based on control system analysis using Hurwitz stability criterion was carried out with 3-DOF mode in three orthogonal x, y and z direction.

4. Conclusion

From the results presented, it was observed that all the determinant values, Δ_i's have negative real parts i.e., $\Delta_i{}'s\ > 0$ which confirmed the stability of the system.

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