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A Modified Population Mean Estimator in Two-Phase Sampling

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Abstract
An exponential-type estimator of population mean in two phase sampling is an improvement of the class of ratio estimator of finite population mean. In real life situation complete information about population mean are usually unavailable which make ratio estimation impracticable. In this study, we proposed a modified exponential-type estimator that can be suitable when population mean of auxiliary variable is unknown in two phase sampling. The proposed estimator was derived using exponential as an improvement strategy. Taylor series expansion up to second degree approximation was used to derive the biases and Mean Square Error (MSE) of the proposed estimator. Efficiency of the proposed estimator was examined. The data sets were used to validate the proposed estimator and their percentage relative efficiency was obtained. The proposed estimator was compared with the exiting estimators. The three proposed estimators are unbiased and efficient. The Mean Square Error (MSE) for the proposed estimators are 0.08037896, 0.1383101 and 7.953746 for the three data set used respectively. The proposed exponential-type estimators of population mean is efficient for estimating population mean in two-phase sampling.

Keywords: *Population mean, Two-phase sampling, Mean square error, Estimator*

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1. Introduction

The use of auxiliary information has been used in practice to increase the efficiency of estimators. This information is generally used in ratio, product and regression type estimators to estimate the population mean of the study variable. When the correlation between the study variable and the auxiliary variable is positive, the ratio estimation method is used. On the other hand, if the correlation is negative, the method of estimating the product is preferred. Some research works such as Searls (1964), Chand (1975), Kiregyera (1984), Murthy (1967) and Singh et al. (2013) have been carried out in ratio, product and regression using one or two auxiliary variables. In this study, a class of relative exponential estimators using two auxiliary variables is considered to estimate a finite population mean for the variable of interest. We have considered several special estimators of the suggested estimators, comparisons between traditional multivariate ratio estimators and estimators proposed by Singh et al. (2013). With the proposed family of estimator using the information two variables are considered. There is a lot of work in which auxiliary information is used to improve the precision of the estimator. Furthermore, on the same pattern, Murthy (1967) proposed a product estimator $\bar{y}_p = \bar{y}\frac{\bar{x}}{\bar{X}}$ to estimates population mean $(\bar{y})$. The product estimator is more efficient than the mean per unit whenever estimator $p < \frac{C_x}{2C_y}$.

The auxiliary information is frequently used to increase precision of the population estimated by taking advantage of the correlation between the study variable and the auxiliary variable. Several authors including Kadilar and Cingi (2006) and Gupta and Shabbir (2010) have proposed different estimators by utilizing information on the auxiliary variable for estimation of population mean. For ratio estimators in sampling theory, population information of the auxiliary variable, such as Coefficient of variable, Coefficient of skewness is often used to increase the efficiency of the estimation for a population mean. Murthy (1967), Chand (1975), Cochran (1977), Prasad and Koseff

(1989), Sen (1993), Upadhyaya and Singh (1999), Singh and Tailor (2005), Singh *et al* (2004), Koyuncu and Kadilar (2009), Lu and Yan (2014), Khan (2016), Olayiwola *et al*. (2020), among others used the population information of the auxiliary variable to increase precision. This study combined the interactive effect of the coefficient of Kurtosis and median of the multiple auxiliary variables to improve the ratio estimators.

2. Materials and methods

Consider a finite population comprises of N units. We draw a sample of size n from the population by using simple random sampling without replacement (SRSWOR). Let y and x be the study and the auxiliary variables of the characteristics y_1 and x_1 respectively for the ith unit. Let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_1$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_1$ be the sample means and $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_1$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_1$ be the population mean. Let $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_1 - \bar{y})^2$ and $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_1 - \bar{x})^2$ be the sample variances. Also, $S_Y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_1 - \bar{Y})^2$ and $S_X^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_1 - \bar{X})^2$ be the population variance. Let p be the correlation coefficient between y and x. Let $c_y = \frac{s_y}{\bar{y}}$ and $c_x = \frac{s_x}{\bar{x}}$ be the coefficient of variation of y and x respectively. The usual unbiased estimator to estimate the population mean of the study variables is

$$T_0 = \bar{y} \quad (1)$$

The variance of the estimator $\bar{y}$ up to first order of approximation is given by

$$Var(T_0) = \bar{y}^2 f_1 C_y^2 \quad (2)$$

The usual ratio in two phase sampling and their mean square error are given by

$$T_1 = \frac{\bar{y}}{\bar{x}}\bar{x}' \quad (3)$$

$$MES(T_1) = \bar{Y}^2\left[f_1 C_y^2 + f_3\left(C_x^2 - 2\rho_{yx} C_y C_x\right)\right] \quad (4)$$

Searls (1964), proposed an estimation procedure for population mean using known knowledge of the coefficient of variation of the auxiliary variable.

$$\bar{\boldsymbol{y}}_1 = \boldsymbol{a}\bar{\boldsymbol{y}} \quad (5)$$

$$var(\bar{\boldsymbol{y}}_1) = (\boldsymbol{1} - \boldsymbol{B})\boldsymbol{f_1}\bar{\boldsymbol{Y}}^2\boldsymbol{C_y^2} \quad (6)$$

where $\boldsymbol{a} = \{\boldsymbol{1} + \boldsymbol{f_1}\bar{\boldsymbol{Y}}^2\boldsymbol{C_y^2}\}$ and $\boldsymbol{B} = \boldsymbol{f_1}\bar{\boldsymbol{Y}}^2\boldsymbol{C_y^2}$

Chand (1975), proposed the following chain ratio-type estimator in double sampling by incorporating the knowledge of two auxiliary variables, the suggested estimator is given by

$$T_2 = \frac{\bar{y}}{\bar{x}}\frac{\bar{x}'}{\bar{z}'}\bar{z} \quad (7)$$

The mean square error of the suggested estimators is given as

$$MSE(T_2) = \bar{y}^2\left[f_1 C_y^2 + f_3\left(C_x^2 - 2p_{yx}C_y C_x\right) + f_2\left(C_z^2 - 2p_{yz}C_y C_z\right)\right] \quad (8)$$

Kiregyera (1984), suggested the following chain-type exponential estimators in two phase sampling, the suggested estimators are given as

$$\bar{y}_2 = \frac{\bar{y}}{\bar{x}}\left[\bar{x}' + b_{yx}(\bar{z} - \bar{z}')\right] \quad (9)$$

$$\bar{y}_3 = \bar{y} + b_{yx}\left[(\bar{x}' - \bar{x}) - b_{xz}(\bar{z} - \bar{z}')\right] \quad (10)$$

The mean square error of the suggested estimators up to first order of approximation are given as following

$$MSE(\bar{y}_2) = \bar{Y}^2\left[f_1 C_y^2 + f_3 C_x\left(C_x - 2p_{yx}C_y\right) + f_2 p_{xz} C_x\left(p_{xz}C_x - 2p_{yx}C_y\right)\right] \quad (11)$$

$$MSE(\bar{y}_3) = \bar{Y}^2 C_y^2\left[f_2 p_{yx}p_{xz}\left(p_{yx}p_{xz} - 2p_{yz}\right) + f_1 - f_3 p_{yx}^2\right] \quad (12)$$

Singh *et al*. (2013), recommended a class of exponential chain ratio-product type estimator for estimating population mean using two auxiliary variables as follow

$$T_3 = \bar{y}\left[\alpha\exp\left(\frac{\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}}{\bar{x}'\frac{\bar{Z}}{\bar{z}'} + \bar{x}}\right) + \beta\exp\left(\frac{\bar{x} - \bar{x}'\frac{\bar{Z}}{\bar{z}'}}{\bar{x} + \bar{x}'\frac{\bar{Z}}{\bar{z}'}}\right)\right] \quad (13)$$

where α and β are suitable chosen constants, such that $\alpha + \beta = 1$

The minimum mean square error of the suggested estimator is given as follow

$$MSE(T_3) = \bar{Y}^2 C_y^2\left[f_1 - \frac{\left(p_{yx}f_3 C_x + p_{yz}f_2 C_z\right)^2}{\left(f_3 C_x^2 + f_2 C_z^2\right)}\right] \quad (14)$$

where the optimum value of α is $\alpha_{opt} = \frac{1}{2} + \frac{(p_{yx}f_3 C_{xy} + p_{yz}f_2 C_{zy})}{f_3 C_x^2 + f_2 C_z^2}$

Lu and yan (2014), proposed family of ratio estimators of a class of multivariate ratio estimators using information of two auxiliary variables as follows:

$$\bar{y}_4 = w_1\bar{y}\frac{a_1\bar{X}_1 + b_1}{a_1\bar{x}_1 + b_1} + w_2\bar{y}\frac{a_2\bar{X}_2 + b_2}{a_2\bar{x}_2 + b_2} \quad (15)$$

where w_1 and w_2 are weights that satisfy the condition, such that $w_1 + w_2 = 1$, MSE of this estimator is given as follows:

$$MSE(\bar{y}_4) = \frac{1-f}{n}\bar{Y}^2\begin{pmatrix} C_y^2 + w_1^2a_1^2C_{x_1}^2 + w_2^2a_2^2C_{x_2}^2 - 2w_1a_1p_{yx_1c_yc_{x_1}} \\ -2w_2a_2p_{yx_2c_yc_{x_2}} + 2w_1w_2a_1a_2p_{x_1x_2}C_{x_1}C_{x_2} \end{pmatrix} \quad (16)$$

Khan (2016), suggested a ratio chain-type exponential estimator for finite population mean of the study variable y, given by

$$T_4 = \bar{y}\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right)^{k_1} + k_2\left[\bar{x}' exp\left(\frac{\bar{Z} - \bar{z}'}{\bar{Z} + \bar{z}'}\right) - \bar{x}\right] \quad (17)$$

where k_1 and k_2 are the unknown constants, whose value is to be determined for optimality conditions.

Hence, the mean square error up to first order of approximation, given as

$$MSE(T_4) = \bar{Y}^2C_y^2\left[f_1 - f_3\rho^2{}_{xy} - \frac{f_2(2\rho_{xy}C_x - \rho_{yz}C_z)^2}{4C_x(C_x - \rho_{xz}C_z) + C_z^2}\right] \quad (18)$$

Olayiwola *et al*. (2020), proposed a class of ratio estimators of a finite population mean using two auxiliary variables under two-phase sample scheme, given as

$$T_5 = \bar{y}\left[a_1\frac{a_1\bar{x}' + b_1}{a_1\bar{x} + b_1} + a_2\frac{a_2\bar{x}' + b_2}{a_2\bar{x} + b_2}\right] \quad (19)$$

Hence, the mean square error up to first order of approximation, given as

$$MSE(T_5) = \bar{Y}^2\begin{bmatrix} f_1\begin{pmatrix} a_1^2Q_1^2C_{x_1}^2 + a_1^2Q_2^2C_{x_2}^2 + a_2^2C_y^2 - 2a_1^2Q_1\rho_{yx_1}C_yC_{x_1} \\ +2a_1a_2Q_1Q_2\rho_{x_1x_2} - 2a_1a_2Q_1\rho_{yx_1}C_yC_{x_1} \\ -2a_1a_2Q_2\rho_{yx_2}C_yC_{x_2} + 2a_1a_2C_y^2 \\ -2a_2^2Q_2\rho_{yx_2}C_yC_{x_2} \end{pmatrix} \\ +f_2\left(a_1^2Q_1^2C_{x_1}^2 + a_1^2Q_2^2C_{x_2}^2 + 2a_1a_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}\right) \end{bmatrix} \quad (20)$$

Proposed estimator

Having studied the Olayiwola *et al*. (2020), the following estimators for estimating finite population mean are proposed:

$$T_7 = \bar{y}\left[\beta_1\, exp\left(\frac{a_1\bar{x}_1' + b_1}{a_1\bar{x}_1 + b_1}\right) + \beta_2\, exp\left(\frac{a_2\bar{x}_2' + b_2}{a_2\bar{x}_2 + b_2}\right)\right] \quad (21)$$

Bias and MSE of estimator T_7

$\bar{y} = (1 + e_0), \bar{x}_1' = (1 + e_1')\bar{X}_1, \bar{x}_1 = (1 + e_1)\bar{X}, \bar{x}_2 = (1 + e_2)\bar{X}_2, \bar{x}_2' = (1 + e_2')\bar{X}_2$

$\boldsymbol{E(e_0) = E(e_1) = (e_2) = E(e_1') = E(e_2') = 0}$

$\boldsymbol{E(e_0^2) = \theta_1C_y^2, E(e_1^2) = \theta_1C_{x_1}^2, E\left(e'^2_1\right) = \theta_2C_{x_1}^2, E(e_2^2) = \theta_1C_{x_2}^2, E\left(e'^2_2\right) = \theta_2C_{x_2}^2}$

$\boldsymbol{E(e_0e_1) = \theta_1\rho_{y_{x_1}}C_yC_{x_1}, E(e_0e_1') = \theta_2\rho_{y_{x_1}}C_yC_{x_1}, E(e_0e_2) = \theta_1\rho_{y_{x_2}}C_yC_{x_2}, E(e_0e_2') = \theta_2\rho_{y_{x_2}}C_yC_{x_2},}$

$\boldsymbol{E(e_1e_1') = \theta_2C_{x_1}^2, E(e_1e_2) = \theta_1\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2}, E(e_1'e_2') = \theta_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2}, E(e_1e_2') = \theta_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2},}$

$\boldsymbol{E(e_1'e_2) = \theta_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2},}$

$$= \bar{y}(1 + e_0)\begin{bmatrix} \beta_1\left[1-\phi_1 e_1+\phi_1 e_1'+\frac{3}{2}\phi_1^2 e_1^2+\frac{1}{2}\phi_1^2 e_1'^2-2\phi_1^2 e_1 e_1'\right] \\ +\beta_2\left[1-\phi_2 e_2+\phi_2 e_2'+\frac{3}{2}\phi_2^2 e_2^2+\frac{1}{2}\phi_2^2 e_2'^2-2\phi_2^2 e_2 e_2'\right]\end{bmatrix} \quad (22)$$

$$E(T_7-\bar{Y})_1 = \bar{y}\left[\beta_1\left[\theta_2\phi_1\rho_{y_{x_1}}C_yC_{x_1}-\theta_1\phi_1\rho_{y_{x_1}}C_yC_{x_1}\right] + \beta_2\left[\theta_2\phi_2\rho_{y_{x_2}}C_yC_{x_2}-\theta_1\phi_2\rho_{y_{x_2}}C_yC_{x_2}\right]\right] \quad (23)$$

$$E(T_7-\bar{Y})_{11} = \bar{Y}\left[\beta_1\left[-\theta_1\phi_1\rho_{y_{x_1}}C_yC_{x_1}\right] + \beta_2\left[\theta_1\phi_2\rho_{y_{x_2}}C_yC_{x_2}\right]\right] \quad (24)$$

$$T_7 = \bar{Y}\begin{bmatrix}\beta_1\left[1-\phi_1 e_1+\phi_1 e_1'+\frac{3}{2}\phi_1^2 e_1^2+\frac{1}{2}\phi_1^2 e'^2_1-2\phi_1^2 e_1 e_1'+e_0-\phi_1 e_0 e_1+\phi_1 e_0 e_1'\right]+ \\ \beta_2\left[1-\phi_2 e_2+\phi_2 e_2'+\frac{3}{2}\phi_2^2 e_2^2+\frac{1}{2}\phi_2^2 e'^2_2-2\phi_2^2 e_2 e_2'+e_0-\phi_2 e_0 e_2+\phi_2 e_0 e_2'\right]\end{bmatrix} \quad (25)$$

$$Bias(T_7)_1 = \bar{Y}\begin{bmatrix}\beta_1\left[(\theta_2-\theta_1)\rho_{y_{x_1}}C_yC_{x_1}-\frac{3}{2}(\theta_2-\theta_1)\phi_1^2C_{x_1}^2\right] \\ +\beta_2\left[(\theta_2-\theta_1)\rho_{y_{x_2}}C_yC_{x_2}-\frac{3}{2}(\theta_2-\theta_1)\phi_2^2C_{x_2}^2\right]\end{bmatrix} \quad (26)$$

$$Bias(T_7)_{11} = \bar{Y}\left[\beta_1\left[\frac{1}{2}(3\theta_1+\theta_2)\phi_1^2C_{x_1}^2-\theta_1\phi_1\rho_{y_{x_1}}C_{y_{x_1}}\right]+\beta_2\left[\frac{1}{2}(3\theta_1+\theta_2)\phi_2^2C_{x_2}^2-\theta_1\phi_2\rho_{y_{x_2}}C_yC_{x_2}\right]\right] \quad (27)$$

$$MSE(T_7)_I = \bar{Y}^2\begin{bmatrix}\beta_1^2\left[\theta_1C_y^2-(\theta_2-\theta_1)\phi_1^2C_{x_1}^2+2(\theta_2-\theta_1)\phi_1\rho_{yx_1}C_yC_{x_1}\right] \\ +\beta_2^2\left[\theta_1C_y^2-(\theta_2-\theta_1)\phi_2^2C_{x_2}^2+2(\theta_2-\theta_1)\phi_2\rho_{yx_2}C_yC_{x_2}\right]+ \\ \beta_1\beta_2\begin{bmatrix}\theta_1C_y^2+(\theta_2-\theta_1)\phi_2\rho_{y_{x_2}}C_yC_{x_2}+(\theta_2-\theta_1)\phi_1\rho_{yx_1}C_yC_{x_1} \\ -(\theta_2-\theta_1)\phi_1\phi_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2}\end{bmatrix}\end{bmatrix} \quad (28)$$

$$MSE(T_7)_{II} = \bar{Y}^2\begin{bmatrix}\beta_1^2\left[\theta_1C_y^2+(\theta_2-\theta_1)\phi_1^2C_{x_1}^2-2\theta_1\phi_1\rho_{y_{x_1}}C_yC_{x_1}\right] \\ +\beta_2^2\left[\theta_1C_y^2+(\theta_2-\theta_1)\phi_2^2C_{x_2}^2-2\theta_1\phi_2\rho_{y_{x_2}}C_yC_{x_2}\right]+ \\ \beta_1\beta_2\left[\theta_1C_y^2-\theta_1\phi_2\rho_{y_{x_2}}C_yC_{x_2}-\theta_1\phi_1\rho_{y_{x_1}}C_yC_{x_1}+\theta_1\phi_1\phi_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2}\right]\end{bmatrix} \quad (29)$$

$$\frac{\partial MSE(T_7)_1}{\partial\beta_1} = 2\beta_1\left[\theta_1C_y^2-(\theta_2-\theta_1)\phi_1^2C_{x_1}^2+2(\theta_2-\theta_1)\phi_1\rho_{y_{x_1}}C_yC_{x_1}\right]+ \beta_2\left[\theta_1C_y^2+(\theta_2-\theta_1)\phi_2\rho_{y_{x_2}}C_yC_{x_2}+(\theta_2-\theta_1)\phi_1\rho_{y_{x_1}}C_yC_{x_1} - (\theta_2-\theta_1)\phi_1\phi_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2}\right]$$

$$\beta_1 = \frac{-\beta_2\left[\theta_1C_y^2+(\theta_2-\theta_1)\phi_2\rho_{y_{x_2}}C_yC_{x_2}+(\theta_2-\theta_1)\phi_1\rho_{y_{x_1}}C_yC_{x_1}-(\theta_2-\theta_1)\phi_1\phi_2\rho_{x_{1_{x_2}}}C_{x_1}C_{x_2}\right]}{2\left[\theta_1C_y^2-(\theta_2-\theta_1)\phi_1^2C_{x_1}^2+2(\theta_2-\theta_1)\phi_1\rho_{y_{x_1}}C_yC_{x_1}\right]} \quad (30)$$

$$\frac{\partial MSE(T_7)_1}{\partial \beta_2} = 2\beta_2\left[\theta_1 C_y^2 - (\theta_2-\theta_1)\phi_2^2 C_{x_2}^2 + 2(\theta_2-\theta_1)\phi_2\rho_{y_{x_2}} C_y C_{x_2}\right]$$
$$+\beta_1\left[\theta_1 C_y^2 - (\theta_2-\theta_1)\phi_2\rho_{y_{x_2}} C_y C_{x_2} + (\theta_2-\theta_1)\phi_1\rho_{y_{x_1}} C_y C_{x_1} - (\theta_2-\theta_1)\phi_1\phi_2\rho_{x_{1_{x_2}}} C_{x_1} C_{x_2}\right]$$

$$\beta_2 = \frac{-\beta_1\left[\theta_1 C_y^2 + (\theta_2-\theta_1)\phi_2\rho_{y_{x_2}} C_y C_{x_2} + (\theta_2-\theta_1)\phi_1\rho_{y_{x_1}} C_y C_{x_1} - (\theta_2-\theta_1)\phi_1\phi_2\rho_{x_{1_{x_2}}} C_{x_1} C_{x_2}\right]}{2\left[\theta_1 C_y^2 - (\theta_2-\theta_1)\phi_2^2 C_{x_2}^2 + 2(\theta_2-\theta_1)\phi_2\rho_{y_{x_2}} C_y C_{x_2}\right]} \quad (31)$$

where

$$a = \theta_1 C_y^2 - (\theta_2-\theta_1)\phi_1^2 C_{x_1}^2 + 2(\theta_2-\theta_1)\phi_1\rho_{yx_1} C_y C_{x_1}$$
$$b = \theta_1 C_y^2 - (\theta_2-\theta_1)\phi_2^2 C_{x_2}^2 + 2(\theta_2-\theta_1)\phi_2\rho_{yx_2} C_y C_{x_2}$$
$$c = \theta_1 C_y^2 + (\theta_2-\theta_1)\phi_2\rho_{y_{x_2}} C_y C_{x_2} + (\theta_2-\theta_1)\phi_1\rho_{yx_1} C_y C_{x_1} - (\theta_2-\theta_1)\phi_1\phi_2\rho_{x_{1_{x_2}}} C_{x_1} C_{x_2}$$

Put β1 in equation (30) and β2 in equation (31) in equation (28).

$$MSE(T_7)_{I\min} = \bar{Y}^2\left[\beta_1^2 a + \beta_2^2 b + \beta_1\beta_2 c\right]$$

$$\beta_1 = \frac{-c}{a-b}, \qquad \beta_2 = \frac{-c}{b-c}$$

$$MSE(T_7)_{I\min} = \bar{Y}^2\left[\frac{c^2}{(a-b)^2}a + \frac{c^2}{(b-c)^2}b + \frac{c^3}{(a-b)(b-c)}\right] \quad (32)$$

$$\frac{\partial MSE(T_7)_{11}}{\partial \beta_1} = \bar{Y}^2\begin{bmatrix} 2\beta_1\left[\theta_1 C_y^2 + (\theta_2+\theta_1)\phi_1^2 C_{x_1}^2 - 2\theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1}\right] \\ +\beta_2\left[\theta_1 C_y^2 - \theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2} - \theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1} + \theta_1\phi_1\phi_2\rho_{x_{1_{x_2}}} C_{x_1} C_{x_2}\right]\end{bmatrix}$$

$$\beta_1 = \frac{-\beta_2\left[\theta_1 C_y^2 - \theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2} - \theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1} + \theta_1\phi_1\phi_2\rho_{y_{x_2}} C_y C_{x_2}\right]}{2\left[\theta_1 C_y^2 + (\theta_2+\theta_1)\phi_1^2 C_{x_1}^2 - 2\theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1}\right]} \quad (33)$$

$$\frac{\partial MSE(T_7)_{11}}{\partial \beta_2} = \bar{Y}^2\begin{bmatrix} 2\beta_2\left[\theta_1 C_y^2 + (\theta_2+\theta_1)\phi_2^2 C_{x_1}^2 - 2\theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2}\right] \\ +\beta_1\left[\theta_1 C_y^2 - \theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2} - \theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1} + \theta_1\phi_1\phi_2\rho_{x_{1_{x_2}}} C_{x_1} C_{x_2}\right]\end{bmatrix}$$

$$\beta_2 = \frac{-\beta_1\left[\theta_1 C_y^2 - \theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2} - \theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1} + \theta_1\phi_1\phi_2\rho_{y_{x_2}} C_y C_{x_2}\right]}{2\left[\theta_1 C_y^2 + (\theta_2+\theta_1)\phi_2^2 C_{x_2}^2 - 2\theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2}\right]} \quad (34)$$

where

$$k = \theta_1 C_y^2 + (\theta_2+\theta_1)\phi_1^2 C_{x_1}^2 - 2\theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1}$$
$$l = \theta_1 C_y^2 + (\theta_2+\theta_1)\phi_2^2 C_{x_2}^2 - 2\theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2}$$
$$m = \theta_1 C_y^2 - \theta_1\phi_2\rho_{y_{x_2}} C_y C_{x_2} - \theta_1\phi_1\rho_{y_{x_1}} C_y C_{x_1} + \theta_1\phi_1\phi_2\rho_{y_{x_2}} C_y C_{x_2}$$

$$MSE(T_7)_{II} = \bar{Y}^2[k\beta_1^2 + l\beta_2^2 + 2\beta_1\beta_2 m] \quad (35)$$

Put β_1 in equation (33) and β_2 in equation (34) in equation (29).

$$\beta_1 = \frac{-m}{k-m}, \beta_2 = \frac{-m}{l-m}$$

$$MSE(T_7)_{II\min} = \bar{Y}^2\left[\frac{m^2}{(k-m)^2}k + \frac{m^2}{(l-m)^2}l + 2\frac{m^3}{(k-m)(l-m)}\right] \quad (36)$$

Empirical study

To examine the merit of the suggested estimator, we have considered five natural population data sets. The descriptions of the population are given below.

Data 1: Lu (2014)

$N = 180; n' = 100; n = 70; \bar{Y} = 13.9951; \bar{X}_1 = 27.3981; \bar{X}_2 = 38.7167; C_y = 0.4180; C_{x_1} = 0.4254;$
$C_{x_2} = 0.3339; \rho_{y_{x_1}} = 0.5630; \rho_{y_{x_2}} = 0.5273; \rho_{x_{1}x_2} = -0.2589; \phi_1 = 0.002; \phi_2 = 1.6519$

Data 2: Source: Lu (2014)

$\boldsymbol{N = 180; n' = 90; n = 50; \bar{Y} = 13.9951; \bar{X}_1 = 17.3981; \bar{X}_2 = 28.7167; C_y = 0.4180; C_{x_1} = 0.4254;}$
$\boldsymbol{C_{x_2} = 0.3339; \rho_{y_{x_1}} = 0.5630; \rho_{y_{x_2}} = 0.5273; \rho_{x_{1}x_2} = 0.2589; \phi_1 = 0.002; \phi_2 = 1.6519}$

Data 3: (Source: Data used by Anderson (1958)): (25 families have been observed for the following three variable.) Y: Head length of second son; X_1: Head length of first son; X_2: Head breadth of first sons

$\boldsymbol{N = 25; n' = 10; n = 7; \bar{Y} = 183.84; \bar{X}_1 = 185.72; \bar{X}_2 = 151.12; C_y = 0.0546; C_{x_1} = 0.0526;}$
$\boldsymbol{C_{x_2} = 0.0488; \rho_{y_{x_1}} = 0.7326; \rho_{y_{x_2}} = 0.6430; \rho_{x_{1}x_2} = 0.6837; \phi_1 = 0.002; \phi_2 = 1.6519}$

3. Results and discussion

A class of relative exponential-type estimate for population mean employing two auxiliary variables was suggested under two phase sampling in the study. The proposed estimator's MSE and PRE (T7) was computed, and the improved estimator had the lowest Mean Square Error (MSE). In comparison to other relative estimators, the Percentage Relative Efficiency (PRE) is higher. As a result, the proposed estimator outperforms the existing estimators.

Table 1: MSE and PRE of proposed and existing estimators using data I

Estimators	**MSE**	**PRE**
T_0	0.2987629	100
T_1	0.2825984	105.72
T_2	0.2515201	118.7829
T_3	0.2492032	119.8873
T_4	0.2380672	125.4952
T_5	0.2896271	103.1544
T_7 (proposed)	0.08037896	371.693

Table 2: MSE and PRE of proposed and existing estimators using data 2

Estimators	MSE	PRE
T_0	0.4943169	100
T_1	0.4607903	107.2759
T_2	0.4219426	117.1526
T_3	0.3989339	123.9095
T_4	0.3725462	132.6861
T_5	0.2585298	191.203
T_7 (proposed)	0.1383101	357.3975

Table 3: MSE and PRE of proposed and existing estimators using data 3

Estimators	MSE	PRE
T_0	10.36334	100
T_1	8.457155	122.5393
T_2	5.795437	178.819
T_3	9.927072	104.3947
T_4	6.465406	160.2891
T_5	10.97317	94.44252
T_7 (proposed)	7.953746	130.2951

4. Conclusion

A class of relative exponential-type estimator for population mean employing two auxiliary variables was suggested under two-phase sampling in this study. The proposed estimator's MSE and PRE (T_7) was computed and compared with the existing estimator (T_0-T_5). The results obtained showed that the suggested estimator was more efficient than all the competing estimators of population mean considered in the study due to minimum value of Mean Square Error (MSE) and the maximum value of percentage Relative Efficiency (PRE) of suggested estimator.

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