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Failure Analysis of a Solid Circular Timber Column Under Axial Compression and Bending

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Abstract
In this paper, the failure analysis of a solid circular timber column under axial compression and bending is carried out in accordance with the design provisions of BS 5268: Part 2, 2002. The potential failure modes considered in the reliability estimation were compression and bending. The grade compression and bending stresses parallel to grain used in the failure analysis correspond to the softwood specie of Strength Class C24 obtained from BS 5268: Part 2, 2002. The limit state functions developed based on the two failure modes were solved to obtain the design points on the failure surface using a MATLAB program developed based on First Order Reliability approximation. Sensitivity analysis was carried out on the random variables to evaluate the contribution of each random variable to the reliability levels and the results obtained showed that the reliability indices decreased with increase in slenderness ratio, decreased with increase in axial and lateral load ratio, decreased with increase in length of column, decreased with increase in axial and lateral loads and increased with increase in column diameter considering both compression and bending failure modes. It was also found that the slenderness ratio of column greater than 50 (this corresponds with 240mm column diameter) and axial and lateral load ratio greater than 1.0 may jeopardize the safety of the column. However, the use of adequate column diameter with a lower value of slenderness ratio would enhance the safety of the column.

Keywords: *Failure analysis, Strength class, Circular timber column, Compression, Bending, Slenderness ratio*

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1. Introduction

Structural design of engineered structures must have reasonable safety margins (Afolayan, 2005; Abubakar, 2006; Ranganathan, 1990). The achievement of absolute safety of engineered systems in the presence of uncertainty is unattainable. Consequently, the use of partial safety factor which is based on experience in the conventional design models will not guarantee structural safety. This can be attributed to the fact that the factors that affect the performance of structural systems are uncertain (Afolayan, 2002; Benu and Sule, 2012). The deterministic design of engineered systems using the conventional safety factors has no provisions for evaluating the extent of conservativeness and sometimes leads to inadequate design or design of structures that are not uneconomical (Aguwa and Sadiku, 2011). Recently, the failure on the part of the design engineers to account for uncertainties in the structural design parameters has led to loss of lives and damage of properties worth millions of naira in Nigeria (Chendo and obi, 2015; Oloyede et al., 2010; Sule, 2011). Consideration of statistical variations in civil engineering design parameters therefore, becomes a task of great importance in the process of setting design criteria for engineered structures. Structural engineering design aims at producing structures or structural members that satisfy both ultimate and serviceability requirements when subjected to service load (Melchers, 1999). The limit states are dependent on the strength of materials of the structure or structural component, structural actions and geometric characteristics of the structure or structural component. The design engineer is faced with the problem of how to cater for the variability that occurs in the strength and geometric properties of the engineering materials. Probability and statistics are always known to provide a framework for dealing with uncertainties that are inherent in the design parameters rationally.

In this paper, the failure analysis of a solid timber column of circular cross section under axial compression and bending is carried out in accordance with the design provisions of *BS 5268: Part 2, 2002*, using First Order Reliability approximation. The formulated limit state equations corresponding to the failure conditions in compression and bending were solved to obtain the design points on the failure surface and their corresponding reliability indices using an optimization algorithm coded in MATLAB language.

2. Performance functions

The performance function is defined as the random difference between structural capacity and load effect. In this study, the performance functions are derived in accordance with the provisions of *BS 5268: Part 2, 2002*, for timber structures. The timber column (Fig. 1) considered in this study is pin-ended and has a circular cross-section.

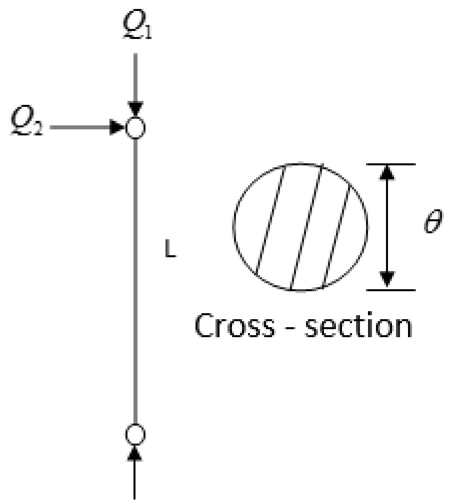

Fig. 1: A pin ended circular timber column under axial compression and bending

2.1 Limit state function in compression

The applied axial stress in column parallel to grain is given by:

$$\sigma_{c,aPar} = \frac{Q_1}{A} = \frac{4Q_1}{\pi\theta^2} \quad (1)$$

Where $\sigma_{c,aPar}$ = applied compressive stress parallel to grain, A= cross- sectional area of the solid circular timber column = $\frac{\pi\theta^2}{4}$, θ = diameter of a solid timber column, Q_1 = ultimate medium term axial load

The ultimate medium-term axial load in compression is given by:

$$Q_1 = P_1(\alpha_1 + 1) \quad (2)$$

where α_1 = axial load ratio (g_{kA}/q_{kA}), P_1 = medium term axial load

Therefore, the failure condition in compression is given by:

$$G(x) = \sigma_{c,perm} - \sigma_{c,aPar} \leq 0 \quad (3)$$

According to BS 5268 (2002),

$$\sigma_{c,Perm} = K_2 K_3 K_8 K_{12}\, \sigma_{c,gPar} \quad \text{for } \lambda \geq 5 \quad (4)$$

Where $\sigma_{c,Perm}$ = permissible compressive stress parallel to grain, $\sigma_{c,gPar}$ = grade compressive stress parallel to grain, K_2 = factor for timber strength class, K_3 = load duration factor, K_8 = load sharing systems factor and K_{12} = compression member stress factor

Substituting Equations (1) and (4) in Equation (3) yields:

$$G(x) = K_2 K_3 K_8 K_{12}\, \sigma_{c,gPar} - \frac{4Q_1}{\pi\theta^2} \quad (5)$$

From structural theory,

$$\lambda = \frac{L}{r} \quad (6)$$

$$r = \sqrt{\frac{I}{A}} \quad (7)$$

where

$$I = \frac{\pi\theta^4}{64} \quad (8)$$

Substituting for I and A in Equation (7) yields:

$$r = \frac{\theta}{4} \quad (9)$$

Substituting for r in Equation (6) yields:

$$\lambda = \frac{4L}{\theta} \quad (10)$$

Where λ = slenderness ratio of a solid circular timber column being considered,
r = radius of gyration of the column section with respect to x or y axis
Multiplying the second term of Equation (5) by L and dividing by same and applying Equation (10) changes Equation (5) to:

$$G(x) = K_2 K_3 K_8 K_{12} \sigma_{c,gpar} - \frac{Q_1 * \lambda}{\pi\theta L} \quad (11)$$

Applying Equation (2), Equation (11) becomes:

$$G(x) = K_2 K_3 K_8 K_{12}\, \sigma_{c,gpar} - \frac{P_1(\alpha_1 + 1)\lambda}{\pi \theta L} \quad (12)$$

P_1 = short term imposed axial load,

$K_2 = 0.6,\ K_3 = 1,\ K_8 = 1,\ K_{12} = 0.799$ (BS 5268, 2002)

Equation (12) is the performance function for a solid circular timber column under axial compression.

2.2 Limit state function in bending

The applied bending stress parallel to grain is given by:

$$\sigma_{m,aPar} = \frac{M}{Z} \quad (13)$$

where

$$M = \frac{Q_2 L^2}{8} \quad (14)$$

and

$$Z = \frac{\pi \theta^3}{32} \quad (15)$$

Applying Equations (14) and (15), Equation (13) becomes:

$$\sigma_{m,aPar} = \frac{4 Q_2 L^2}{\pi \theta^3} \quad (16)$$

where Q_2 = ultimate short-term lateral load

Similarly, the ultimate short-term lateral load is given by:

$$Q_2 = P_2(\alpha_2 + 1) \quad (17)$$

According to BS 5268, Part 2 (2002), the permissible bending stress parallel to grain is given by:

$$\sigma_{m,Perm} = K_2 K_3 K_6 K_7 K_8\, \sigma_{m,gPar} \quad (18)$$

where $\sigma_{m,gPar}$ = grade bending stress parallel to grain, $\sigma_{m,aPar}$ = applied bending stress parallel to grain, K_2, K_3, K_6, K_7, K_8 are moisture content factor, load duration factor, form factor, depth factor, load sharing systems factor respectively, α_2 = lateral load ratio (g_{kL}/q_{kL})

Therefore, the failure condition in bending is given by:

$$G(x) = \sigma_{m,Perm} - \sigma_{m,aPar} \le 0 \quad (19)$$

Applying Equations (16) and (18), Equation (19) becomes:

$$G(x) = K_2 K_3 K_6 K_7 K_8\, f_{m,gPar} - \frac{4 Q_2 L^2}{\pi \theta^3} \quad (20)$$

Again, substituting for Q_2 using Equation (17) changes equation (20) to:

$$G(x) = K_2 K_3 K_6 K_7 K_8\, \sigma_{m,gpar} - \frac{4 P_2(\alpha_2 + 1) L^2}{\pi \theta^3} \quad (21)$$

Applying Equation (10), Equation (21) becomes:

$$G(x) = K_2 K_3 K_6 K_7 K_8\, \sigma_{m,gpar} - \frac{P_2(\alpha_2 + 1) L\, \lambda}{\pi \theta^2} \quad (22)$$

where P_2 = short term lateral load, α_2 = lateral load ratio,

$K_2 = 0.8,\ K_3 = 1,\ K_6 = 1.18,\ K_7 = 1,\ K_8 = 1$

(BS 5268, 2002)

Equation (22) represents the performance function for a solid circular timber column in bending.

3. Materials and methods

The First Order Reliability algorithm coded in MATLAB language was invoked to search for the design points on the failure surface and the corresponding reliability levels. The grade stresses in compression and bending were obtained in accordance with the provisions of BS 5268: Part 2, 2002.

The vector of random variables, $X = (X_1, X_2, ..., X_n)$ are the basic variables having joint probability function given by Equation (23):

$$F_X(X) = P\left(\bigcap_{i=1}^{n} \{X_i \le x_i\}\right) \quad (23)$$

In this study, $F_X(X)$ is assumed to be continuous and differentiable with respect to the basic variables, implying that the probability density of $F_X(X)$ exists. The performance function, $g(X)$ of a structure corresponding to each limit state is dependent on the basic variables. $g(X) > 0$ corresponds to the safe domain of the structure, $g(X) = 0$ corresponds to the failure boundary and $g(X) < 0$ corresponds to the failure state of the structure.

The probability of failure can be defined mathematically as:

$$P_f = P[g(X) \le 0] = \phi(-\beta) \quad (24)$$

where β = reliability index which represents the minimum distance between the origin and the failure surface. It is given by:

$$\beta = \min \{\|X\|\}\ for\ \{X : g(X) < 0\} \quad (25)$$

The statistics of the basic variables are shown in Table 1.

Table 1: Probabilistic models of random variables

S/N	Variables	Probability Distribution	Mean (X)	Standard Deviation (X)	COV (X)
1	P_1	Gumbel	65,000N	1950N	0.030
2	P_2	Gumbel	3.25N/mm	0.975N/mm	0.30
4	L	Normal	3000mm	30mm	0.01
5	θ	Normal	300mm	3mm	0.01
6	$\sigma_{m,gpar}$	Lognormal	7.5N/mm^2	1.125N/mm^2	0.15
7	$\sigma_{c,gpar}$	Lognormal	7.9N/mm^3	1.185N/mm^3	0.15
10	λ	-	-	-	Deterministic
11	α_1	-	-	-	Deterministic
12	α_2	-	-	-	Deterministic

Source: Benu and Sule (2012)

4. Results and discussion

A MATLAB program developed based on First Order Reliability approximation was invoked to estimate the reliability indices. The results of the reliability analysis are shown in Figures 2 to 8.

It can be observed that:

i. The reliability index decreased with increase in slenderness ratio at 3m length of column considering compression and bending failure criteria (Figures 2 and 3). This is because the column becomes unstable as the slenderness ratio increases leading to the reduction of column stiffness. Also, as the load ratio (axial and lateral load ratio) increases, the load carrying capacity of the column reduces thereby reducing the reliability indices of the column.

ii. Using a timber column of slenderness ratio greater than 50 and axial and lateral load ratio greater than 1 may jeopardize the safety of the column (Figures 2 and 3).

iii. The maximum axial load on the column should not exceed 85000N (Figure 7).

iv. The reliability index increased with increase in column diameter at 3m length of column considering compression and bending failure criteria (Figures 4 and 5). This is because the column stiffness increases with increase in column diameter.

v. Reliability index decreased with increase in length of column considering bending failure mode at column length of 3m (Figure 6). This is because the slenderness ratio of the column increases as the length of the column increases thereby making the column vulnerable to failure by buckling.

vi. The axial load ratio has no effect on the reliability indices of a timber column in bending (Figures 3, 5, 6, and 8).

vii. The lateral load ratio has no effect on the reliability indices of a timber column in compression (Figures 5, 4, and 7).

viii. The reliability indices decreased with increase in load ratio for varying values of axial and lateral loads at constant slenderness ratio (lambda = 40) and length of column (3m) considering compression and bending failure modes (Figures 7 and 8). This attribute is expected because as the values of the axial and lateral loads increase the load carrying capacity of the column is expected to reduce thereby reducing the reliability indices.

ix. The timber column is safe considering the average target reliability index of 2.5 (Melchers, 1999) required for timber structures for the two failure modes considered. However, the use of suitable and adequate dimension of the timber column having a lower slenderness ratio will improve the safety of the column (Benu and Sule, 2012).

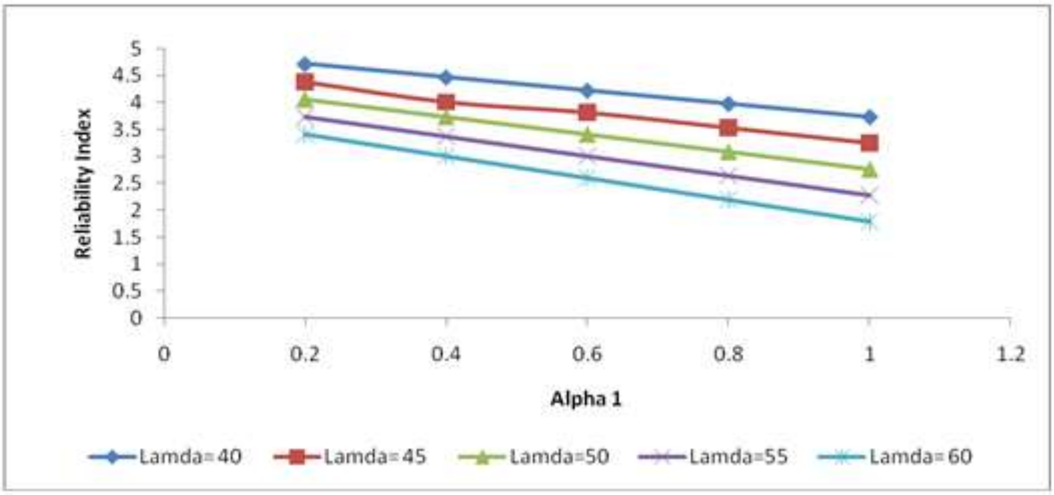

Fig. 2: Reliability index against load ratio (alpha 1) at varying column slenderness ratio for 3m length of column (compression)

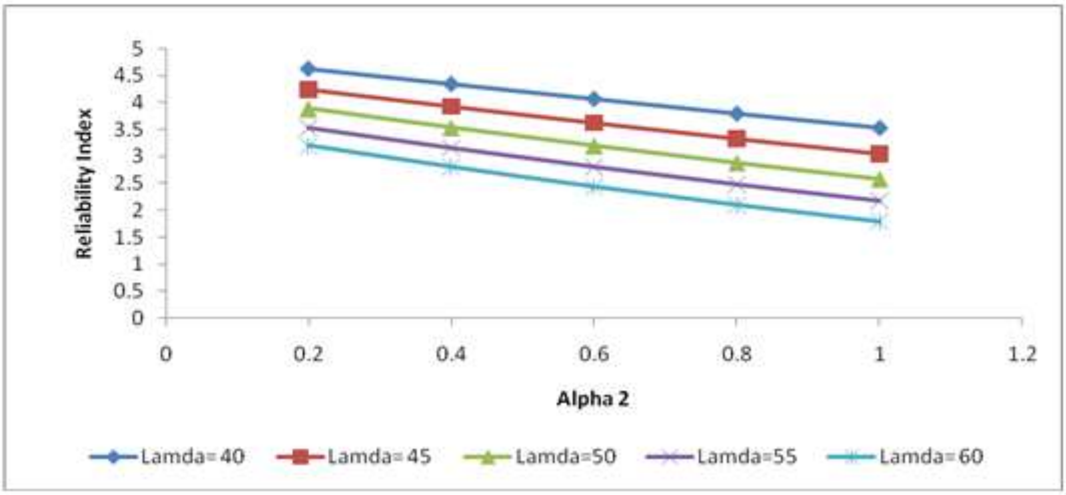

Fig. 3: Reliability index against load ratio (alpha 2), at varying column slenderness ratio for 3m length of column (bending)

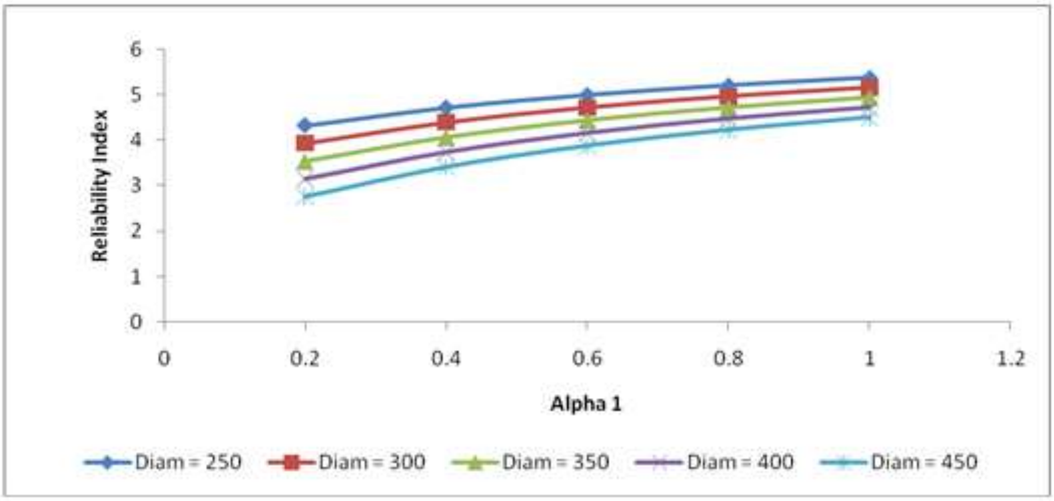

Fig. 4: Reliability index against load ratio (alpha 1) at varying column diameter for 3m length of column (compression)

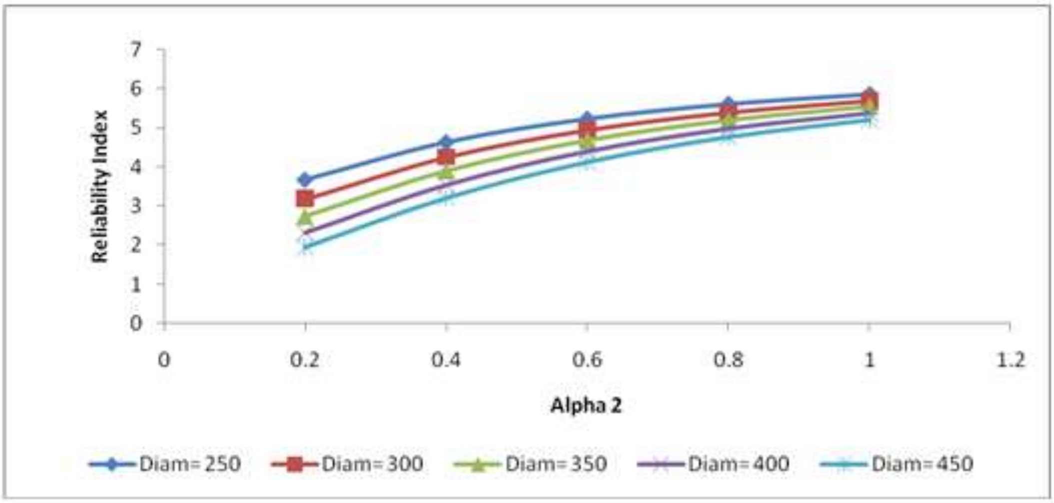

Fig. 5: Reliability index against load ratio (alpha 2) at varying column diameter for 3m length of column (bending)

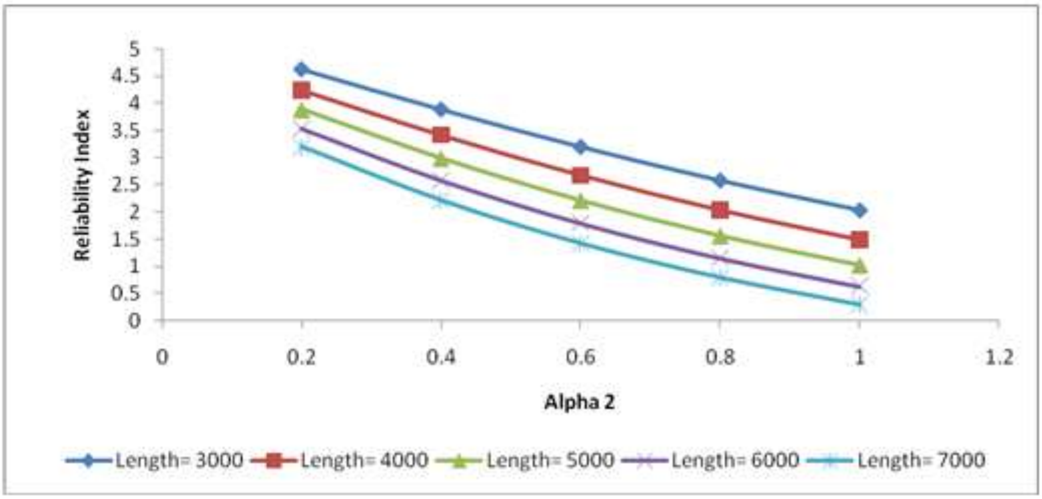

Fig. 6: Reliability index against load ratio (alpha 2) at varying length of column (bending)

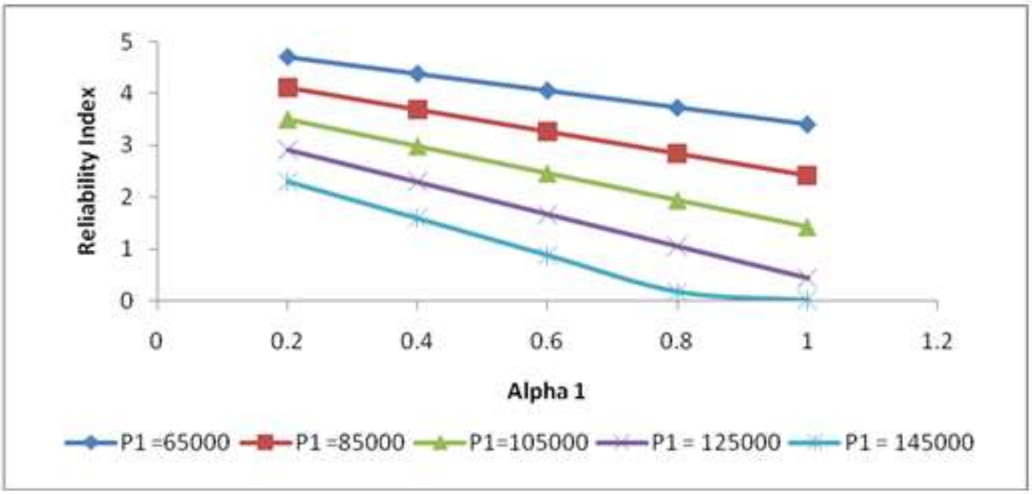

Fig. 7: Reliability index against load ratio (alpha 1), at varying axial load for 3m length of column (compression)

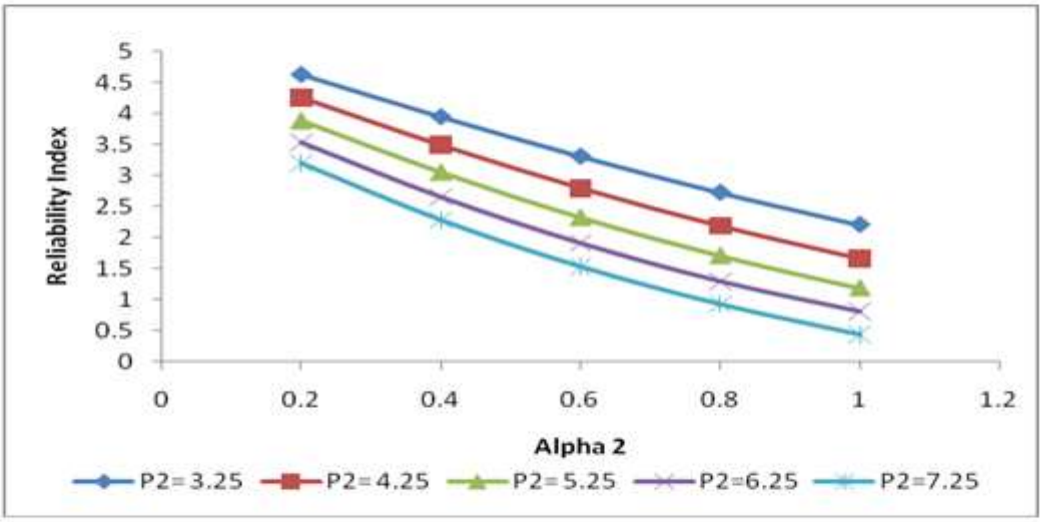

Fig. 8: Reliability index against load ratio (alpha 2), at varying lateral for 3m length of column (bending)

5. Conclusions

The following conclusions were drawn based on the results obtained from the reliability analysis.

i. Reliability index decreased with increase in slenderness ratio, axial and lateral load ratio, length of column and increased with increase in column diameter, considering compression and bending failure modes.
ii. Using a solid circular column of slenderness ratio greater than 50 (240mm column diameter) and axial and lateral load ratio greater than 1.0 may jeopardize the safety of the column.
iii. The axial load ratio has no visible effect on the reliability of a solid circular timber column in bending.
iv. The lateral load ratio has no visible effect on the reliability of a solid circular timber column in compression.
v. Reliability index decreased with increase in axial and lateral load ratio for varying values of axial and lateral loads at constant slenderness ratio (lambda = 40) and length of column (3m), for both compression and bending failure modes.

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