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Reliability Assessment of a Solid Timber Column at Ultimate Limit State

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Abstract
In this paper, the results of safety assessment of a solid timber column of Strength Class C24 subjected to axial compression and bending in accordance with the design requirements of Eurocode 5 (EC5) are discussed. Axial compression, bending and compression-bending interaction were the potential failure criteria considered in the reliability estimation. The design points of the derived performance functions and their corresponding reliability indices were obtained using a MATLAB program developed based on First Order reliability approximation. It was found from the sensitivity analysis carried out on the design parameters that the reliability indices increased with increase in radius of gyration, decreased with increase in load ratio considering the three failure modes considered. The reliability indices were also found to decrease with increase in length of column considering the three failure modes. The reliability indices were also found to decrease with increase in lateral and axial loads for varying lengths of column considering bending and compression-bending failure modes. The design was found to be unsafe at 6m and 7m length of column for bending and compression-bending interaction failure modes for all the load ratios considered. The design was found to be conservative when compared with the target safety index value of 3.8 for 50-year reference period for Reliability Class 2 at ultimate limit state for all the modes considered. However, the choice of adequate and suitable dimensions with a lower slenderness ratio will improve the reliability of the column.

Keywords: *Strength class, Solid timber column, Eurocode 5, Failure criteria, Radius of gyration*

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1. Introduction

The actual loads that act on civil engineering structures cannot be determined with certainty in a deterministic framework due to presence of uncertainties in the design parameters. The conventional civil and structural engineering design models make use of partial safety factors whose choice are empirical and do not have explicit reliability significance. This is because structural problems that occur in real world are stochastic and not deterministic (Afolayan, 2002). The use of traditional partial safety factors may lead to over-design or under-design of structures or structural components due to lack of knowledge of the actual loads that act on the structure (Afolayan and Opeyemi, 2008; Melchers et al., 1999; Sule, 2011; Abubakar, 2006; Akindahunsi and Afolayan, 2009; Ranganathan, 1999). Underestimation of uncertainties in the design parameters has led to the collapse of many buildings in Nigeria leading to loss of many lives and damage of properties worth millions of naira (Chendo and Obi, 2015). The task of the structural engineer is to design and maintain the structure with a view to deferring the failed state of the structure and in this task, he faces the problem inherent in the variability of engineering materials. However, a probabilistic approach always provides a rational way of dealing with such uncertainties that are inherent in structures by using statistical approach. Probabilistic concept may not provide solutions to all issues of structural uncertainties, but it has helped immensely in the reliability assessment of many civil engineering facilities (Abejide, 2014; Abubakar and Edache, 2007).

In this paper, the safety analysis of a solid square timber column subjected to both axial and lateral loading is carried out in accordance with the design requirements of Eurocode 5 based on First Order Reliability approximation. The performance functions were derived based on compression, bending and compression-bending interaction failure modes and were solved to obtain the safety indices using a MATLAB program written based on First Order Reliability approximation.

2. Formulation of limit state functions

The limit state functions were derived in accordance with the design requirements of EC 5 for timber design. The timber column (Fig. 1) considered in this study is pin-ended and has a square cross section.

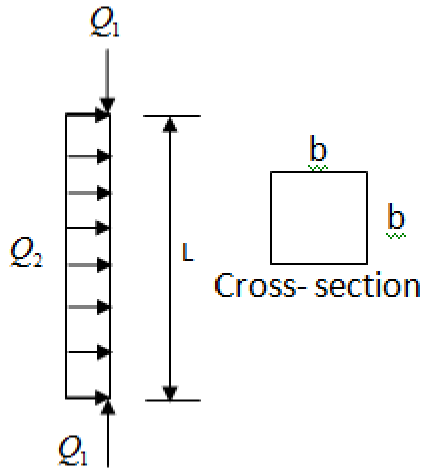

Fig. 1: A pin ended square timber column under compression and bending

2.1 Compression failure criterion

The design compressive stress in column parallel to grain is given by:

$$\sigma_{c,o,d} = \frac{Q_1}{A} = \frac{Q_1}{b^2} \qquad (1)$$

Where $\sigma_{c,o,d}$ = design stress, A = cross-sectional area, b = cross sectional dimension, Q_1 = ultimate medium term axial load

The strength design requirement according to EC 5 is given by:

$$f_{c,o,d} = \frac{K_{\text{mod}}\, f_{c,k}}{\gamma_m} \qquad (2)$$

Where $f_{c,o,d}$ = compressive strength parallel to the grain, K_{mod} = modification factor that accounts for the effect of the duration of load and moisture content, γ_m = partial safety factor for the material property based on EC 5, $f_{c,k}$ = characteristic value of the compressive strength.

Therefore, the failure condition of the column in compression using Equation (1) and (2) is given by:

$$g(x) = f_{c,o,d} - \sigma_{c,o,d} \leq 0 \qquad (3)$$

Substituting Equations (1) and (2) in Equation (3) yields:

$$g(x) = \frac{K_{\text{mod}}\, f_{c,k}}{\gamma_m} - \frac{Q_1}{b^2} \qquad (4)$$

For a solid square timber cross-section, the minimum moment of inertia about x and y axis is:

$$I_{xx} = I_{yy} = I = Ar^2 \qquad (5)$$

The column has a square cross-section. Therefore,

$$I = \frac{b^4}{12} \qquad (6)$$

$$A = b*b \qquad (7)$$

From Equations (5), (6) and (7), r becomes:

$$r = \frac{b}{\sqrt{12}} \qquad (8)$$

From Equations (8),

$$b^2 = 12r^2 \qquad (9)$$

Where: r = radius of gyration about both x and y axis respectively

Using Equation (9), Equation (1) becomes:

$$\sigma_{c,o,d} = \frac{Q_1}{A} = \frac{Q_1}{12r^2} \qquad (10)$$

The ultimate medium-term axial load is given by:

$$Q_1 = P_1(1.35\alpha_a + 1.5) \qquad (11)$$

From Equation (11), Equation (4) becomes:

$$g(x) = \frac{K_{\text{mod}}\, f_{c,k}}{\gamma_m} - \frac{P_1(1.35\alpha_a + 1.5)}{b^2} \qquad (12)$$

Where P_1 = medium term axial load, α_a = axial load ratio in compression

The axial load ratio in compression is defined as:

$$\alpha_a = \frac{g_k}{q_k} \qquad (13)$$

Where g_k = dead load and q_k = medium term-imposed load.

Substituting for b^2 in Equation (12) transforms Equation (12) to:

$$g(x) = \frac{K_{\text{mod}}\, f_{c,k}}{\gamma_m} - \frac{P_1(1.35\alpha_a + 1.5)}{12r^2} \qquad (14)$$

2.2 Bending failure criterion

The applied bending stress parallel to grain is given by:

$$\sigma_{m,o,d} = \frac{M}{Z} \quad (15)$$

The maximum bending moment based on the idealized beam model is given by:

$$M = \frac{Q_2 L^2}{8} \quad (16)$$

From structural theory, the elastic section modulus for a solid square section is given by:

$$Z = \frac{b^3}{6} \quad (17)$$

Also, the ultimate uniformly distributed short-term lateral load is given by:

$$Q_2 = P_2(1.35\alpha_l + 1.5) \quad (18)$$

Where α_l = lateral load ratio in bending, P_2 = short term lateral load.

The lateral load ratio in bending is defined as:

$$\alpha_l = \frac{g_k}{q_k} \quad (19)$$

Substituting for M and Z in Equation (15) using Equations (16), (17) and (18) transforms Equation (15) to:

$$\sigma_{m,o,d} = \frac{0.75P_2(1.35\alpha_l + 1.5)L^2}{b^3} \quad (20)$$

From Equation (8), Equation (20) becomes:

$$\sigma_{m,o,d} = \frac{0.75P_2(1.35\alpha_l + 1.5)L^2}{41.57r^3} \quad (21)$$

Where P_2 = uniformly distributed short term lateral load.

According to EC 5, the design bending strength of a timber column parallel to grain is given by:

$$f_{m,,o,d} = \frac{K_{\text{mod}} f_{m,k}}{\gamma_m} \quad (22)$$

Where: $f_{m,k}$ = Characteristic value of the bending strength.

The performance function in bending is given by:

$$g(x) = f_{m,o,d} - \sigma_{m,o,d} \quad (23)$$

Using Equations (20) and (22), Equation (23) becomes:

$$g(x) = \frac{K_{\text{mod}} f_{m,k}}{\gamma_m} - \frac{0.75P_2(1.35\alpha_l + 1.5)L^2}{b^3} \quad (24)$$

Also, expressing b in terms of r using Equation (8) transforms Equation (24) to:

$$g(x) = \frac{K_{\text{mod}} f_{m,k}}{\gamma_m} - \frac{0.75P_2(1.35\alpha_l + 1.5)L^2}{41.57r^3} \quad (25)$$

2.3 Compression-bending interaction failure criterion

According to EC 5, the compression-bending interaction equation for $\lambda_{rel} < 0.3$ should satisfy the condition of Equation (25) as follows:

$$g(x) = \left(\frac{\sigma_{c,o,d}}{f_{c,o,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{K_m \sigma_{m,z,d}}{f_{m,z,d}} \le 1.0 \quad (26)$$

A square solid square section is considered in this study. Therefore,

$$\begin{Bmatrix} \sigma_{m,y,d} = \sigma_{m,z,d} = \sigma_{m,o,d} \\ f_{m,y,d} = f_{m,z,d} = f_{m,o,d} \end{Bmatrix} \quad (27)$$

Using Equation (27), Equation (26) now becomes:

$$g(x) = \left(\frac{\sigma_{c,o,d}}{f_{c,o,d}}\right)^2 + \frac{2K_m \sigma_{m,o,d}}{f_{m,o,d}} \le 1.0 \quad (28)$$

According to EC5, K_m = 1.0 for a square cross section

K_m = factor that caters for re-distribution of stresses and the effect of in homogeneities of the material in cross-sections.

Substituting for $\sigma_{c,o,d}$, $f_{c,o,d}$, $\sigma_{m,o,d}$ and $f_{m,o,d}$ using Equations (2), (10), (21) and (22), transforms Equation (28) to:

$$g(x) = \left(\frac{\gamma_m P_1(1.35\alpha_a + 1.5)}{12r^2 K_{\text{mod}} f_{ck}}\right)^2 + \frac{0.75\gamma_m P_2(1.35\alpha_l + 1.5)L^2}{41.57r^3 K_{\text{mod}} f_{m,k}} \le 1.0 \quad (29)$$

Using Equation (29), the limit state function for compression-bending interaction is given by:

$$g(x) = 1 - \left[\left(\frac{\gamma_m P_1(1.35\alpha_a + 1.5)}{12r^2 K_{\text{mod}} f_{ck}}\right)^2 + \frac{0.75\gamma_m P_2(1.35\alpha_l + 1.5)L^2}{41.57r^3 K_{\text{mod}} f_{m,k}}\right] \quad (30)$$

3. First order reliability approximation

According to First Order Reliability approximation, the vector $X = (X_1, X_2, ..., X_n)^T$ represents the basic variables whose joint probability function is given by:

$$F_X(X) = P\left(\bigcap_{i=1}^{n}\{X_i \le x_i\}\right) \quad (31)$$

$F_X(X)$ is continuous and differentiable with respect to the basic variables. That is to say that the probability density of $F_X(x)$ exists. The safety margin, $g(x)$ of a structure corresponding to a particular limit state is usually dependent on the basic variables. Graphically, the function $g(x) > 0$ represents safe state of the structure, $g(x) < 0$ represents the unsafe state of the structure and $g(x) = 0$ represents failure surface and it represents the line of demarcation between the safe and unsafe state of the structure.

First order approximation to probability of failure is given by:

$$P_f = P(X \in F) = P(g(X) \le 0 = \int_{g(x)\le 0} dF_X(x) = \phi(-\beta) \quad (32)$$

Where $g(x) \le 0$ represents the domain of failure

Where: β = geometric reliability index and it represents the minimum distance between the origin and the limit state surface.

It is given by:

$$\beta = \min \{\|X\|\} \; for \{X : g(X) < 0\} \quad (33)$$

The values of the design variables that minimize the reliability index, β subject to $g(x) = 0$ are obtained through an optimization algorithm. The statistics of the basic variables are shown in Table 1. The characteristic values of bending and compressive strength parallel to grain used in this study correspond to softwood specie of Strength Class C24 obtained from EN 338 (2009).

Table 1: Probabilistic models of random parameters

S/N	Basic Variable	Mean	Standard Deviation	Variation Coefficient	Probability Distribution
1	P_1	65,000N	1950N	0.030	Gumbel
2	P_2	3.25N/mm	0.975N/mm	0.30	Gumbel
3	K_{mod}	0.90	0.135	0.15	Lognormal
4	L	3000mm	30mm	0.01	Normal
6	$f_{m,k}$	24N/mm^2	3.6N/mm^2	0.15	Lognormal
7	$f_{c,k}$	21N/mm^2	3.15N/mm^2	0.15	Lognormal
8	γ_m	1.30	0.195	0.15	Lognormal
10	r	86.6mm	4.33mm	0.05	Normal
11	α	0.2-1.0	-	-	Deterministic
12	b	300mm	3mm	0.01	Normal

Source: Benu and Sule (2012); Wilson et al. (2019)

4. Results and discussion

Reliability indices for the various failure criteria considered in the study were obtained using a First Order reliability approximation based MATLAB program. The results of the sensitivity analysis carried out on the random variables are shown in Figures 2 to 13 respectively.

From the various graphs, it can be observed that:

i. Reliability index decreased with increase in load ratio considering compression, bending and compression-bending failure modes (Fig. 2). This is because the increase in the value of load ratio led to the reduction in the load bearing capacity of the column.

ii. The reliability index decreased with increase in load ratio for 3m, 4m, 5m, 6m and 7m length of column considering bending and compression-bending failure modes (Fig. 3 and 4). This is because as the column length increases, the slenderness ratio of the column also increases thereby increasing the vulnerability of the column to failure by buckling. This scenario reduces the load carrying capacity of the column and this reduces the reliability index. Also, increase in load ratio reduces the load carrying capacity of the column.

iii. Reliability index increased with increase in value of radius of gyration considering compression, bending and compression-bending failure modes. This is because as the radius of gyration increases, the slenderness ratio of the column reduces and this enhances the stability of the column (Fig. 5, 6 and 7).

iv. The reliability index decreased with increase in axial load considering compression and compression-bending failure modes (Fig. 8 and 9) and also decreased with increase in lateral load considering bending and compression-bending failure modes (Fig. 10 and 11). This behavior is expected because increasing the axial and lateral loads beyond the load bearing capacity of the column will definitely jeopardize the safety of the column.

v. Reliability index decreased with increase in the cross-sectional dimensions (width or depth) of the column considering compression and bending failure modes (Fig. 12 and 13). This is because as the depth increases, the column stiffness increases thereby enhancing the safety of the column.

vi. The timber column is safe considering the target reliability index of 2.5 (Melchers, 1999) required for timber members for all the failure modes considered. Good choices of column cross-sectional dimensions with a lower value of slenderness ratio will improve the safety of the column (Benu and Sule, 2012).

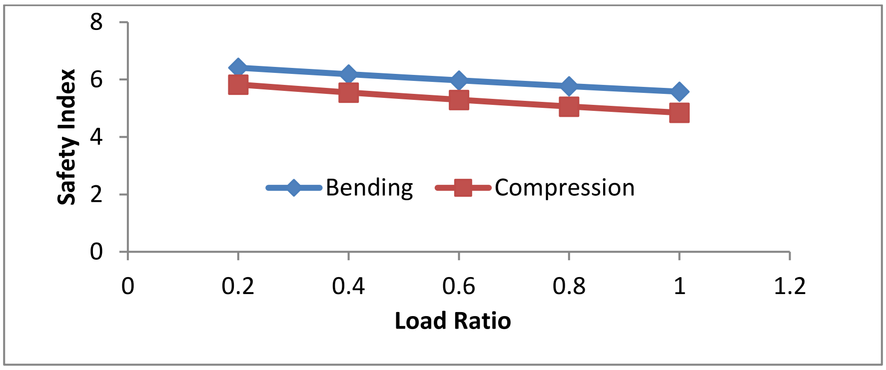

Fig. 2: Safety index against load ratio in bending and compression

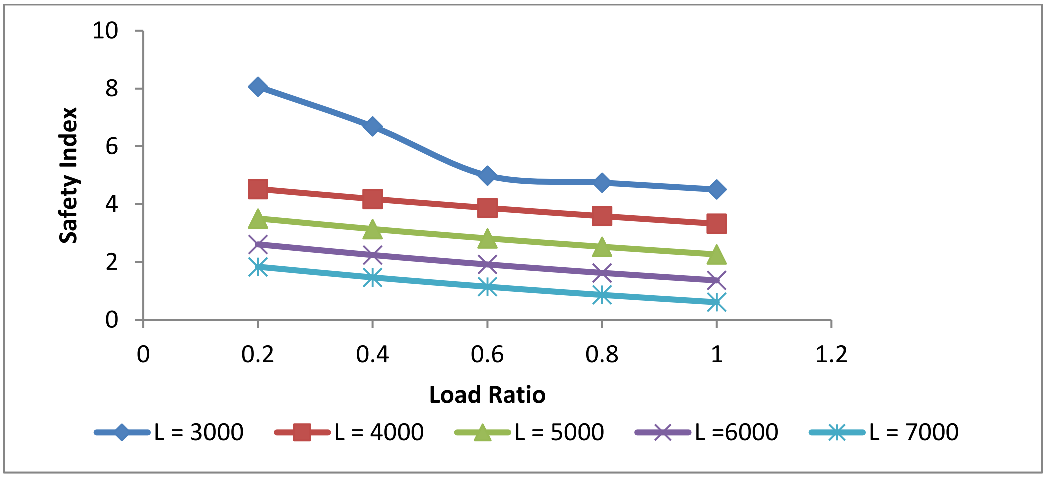

Fig. 3: Safety index against load ratio at varying length of column (compression-bending interaction failure mode)

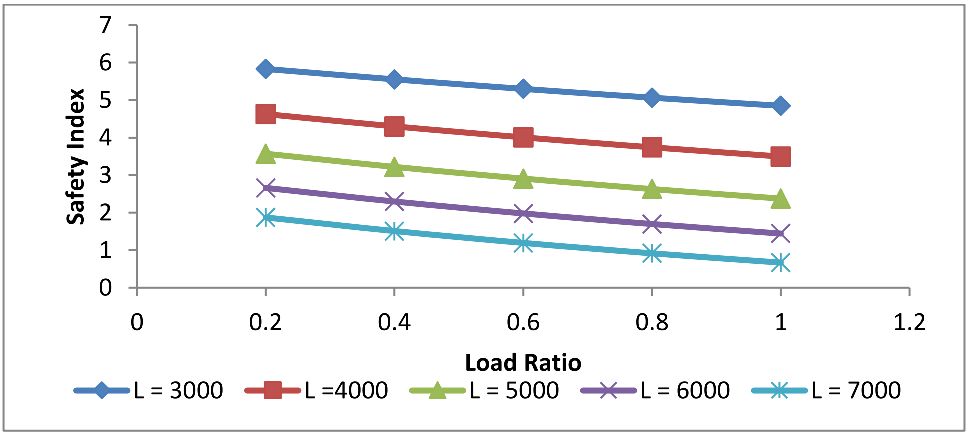

Fig. 4: Safety index against load ratio at varying length of column (bending failure mode)

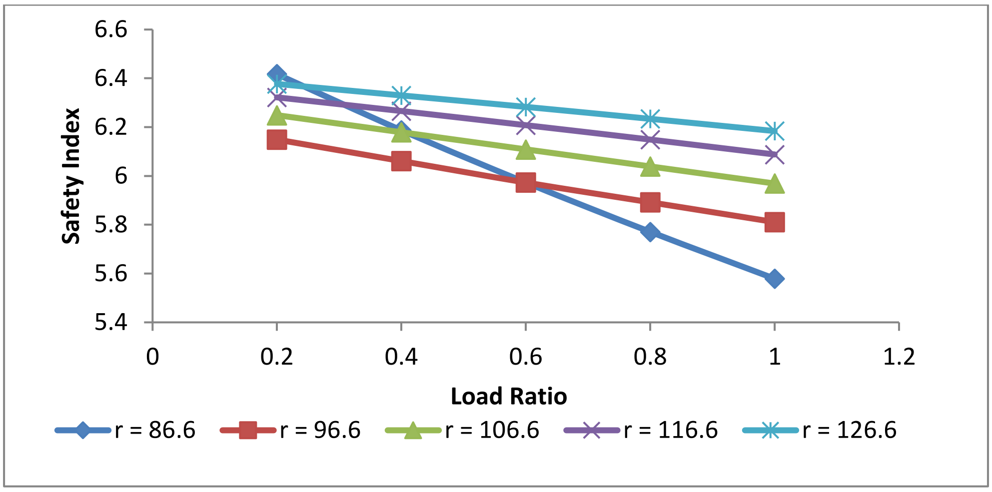

Fig. 5: Safety index against load ratio at varying radius of gyration (compression failure mode)

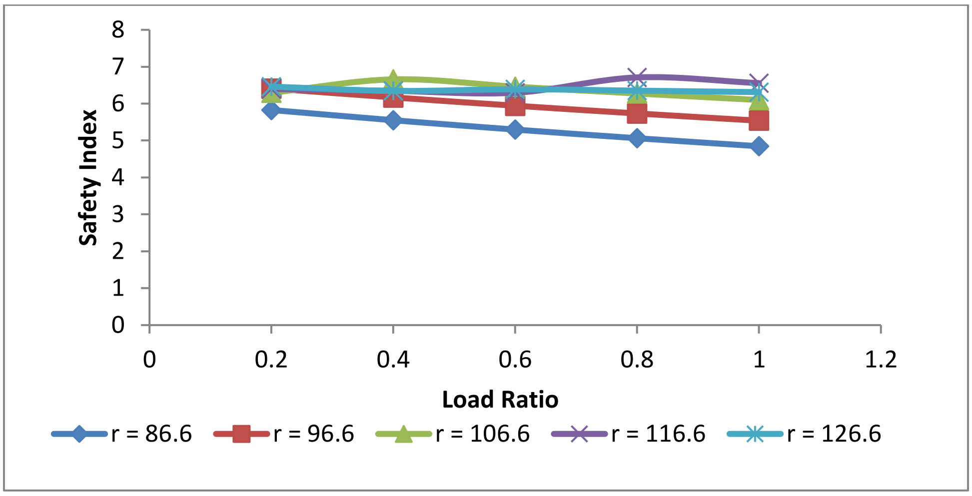

Fig. 6: Safety index against load ratio at varying radius of gyration (bending failure mode)

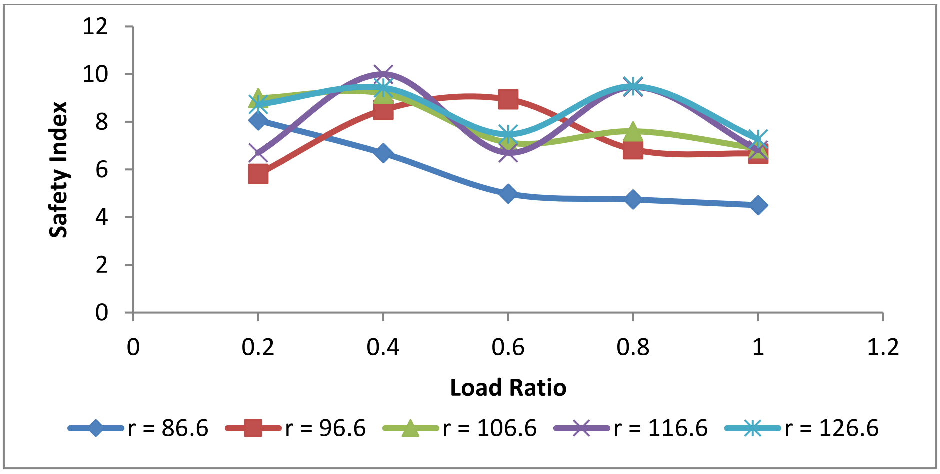

Fig. 7: Safety index against load ratio at varying radius of gyration (compression-bending failure mode)

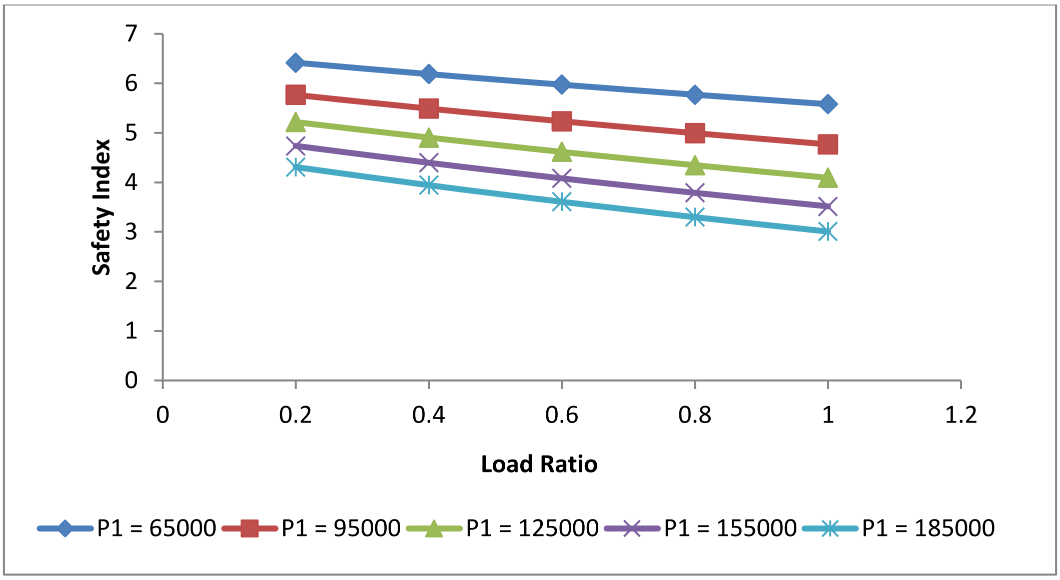

Fig. 8: Safety index against load ratio at varying axial load (compression failure mode)

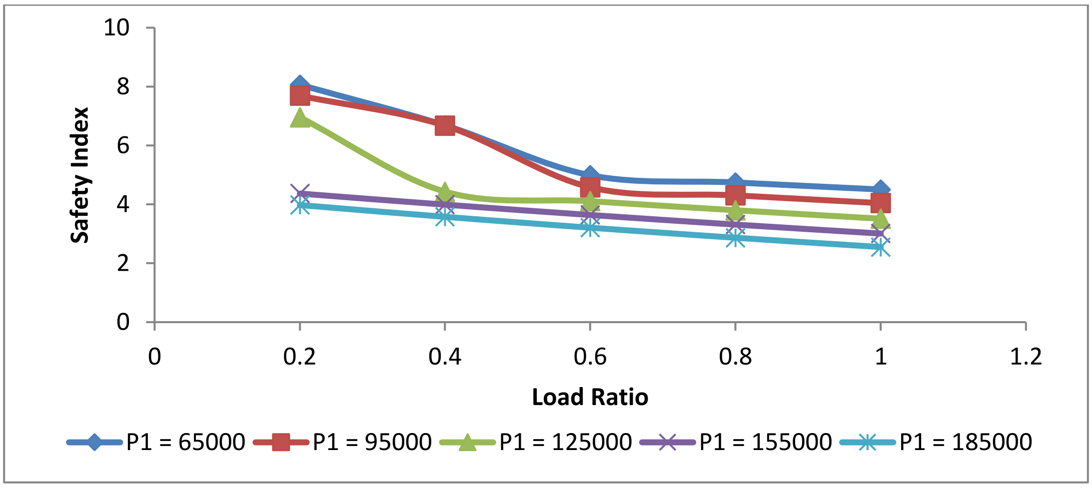

Fig. 9: Safety index against load ratio at varying axial load (compression-bending failure mode)

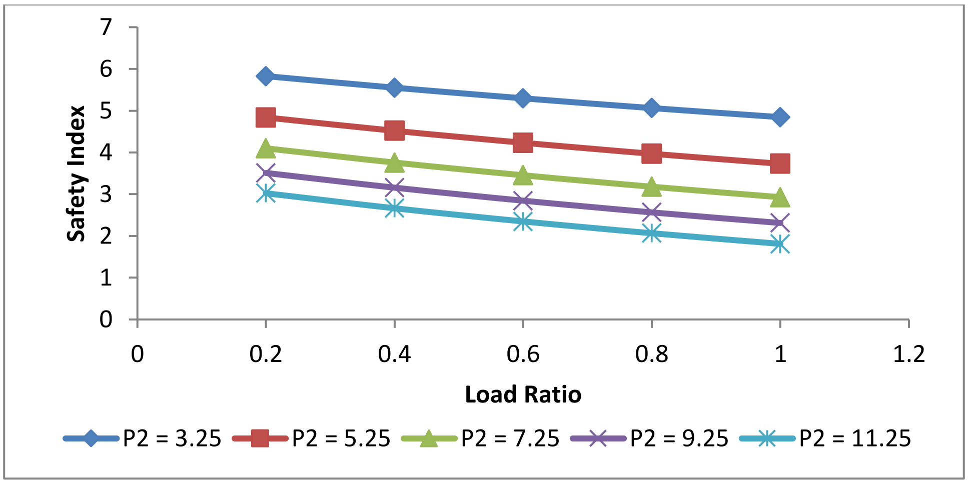

Fig. 10: Safety index against load ratio at varying lateral load (bending failure mode)

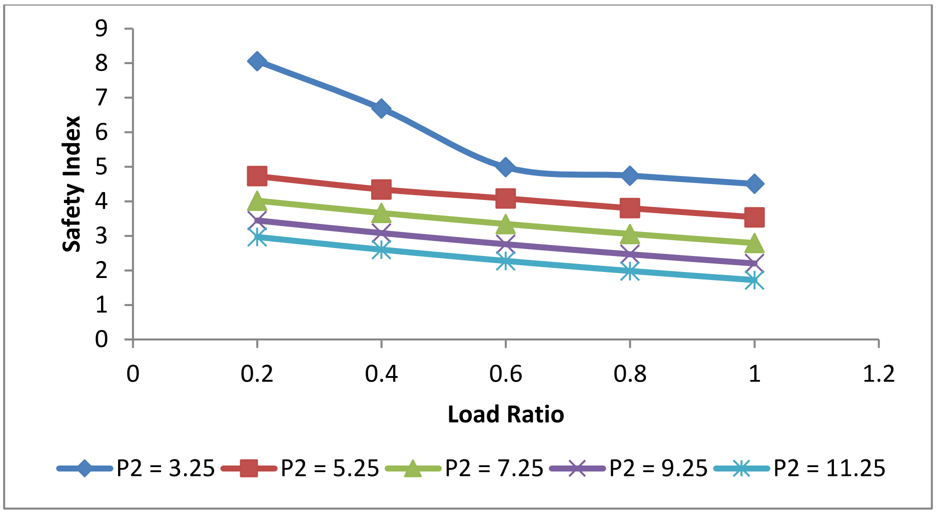

Fig. 11: Safety index against load ratio at varying lateral load (compression-bending failure mode)

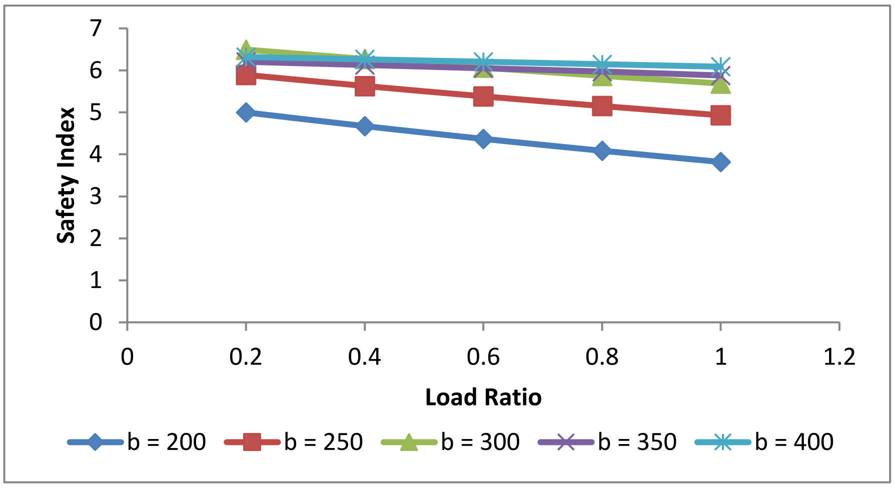

Fig. 12: Safety index against load ratio at varying column depth (compression failure mode)

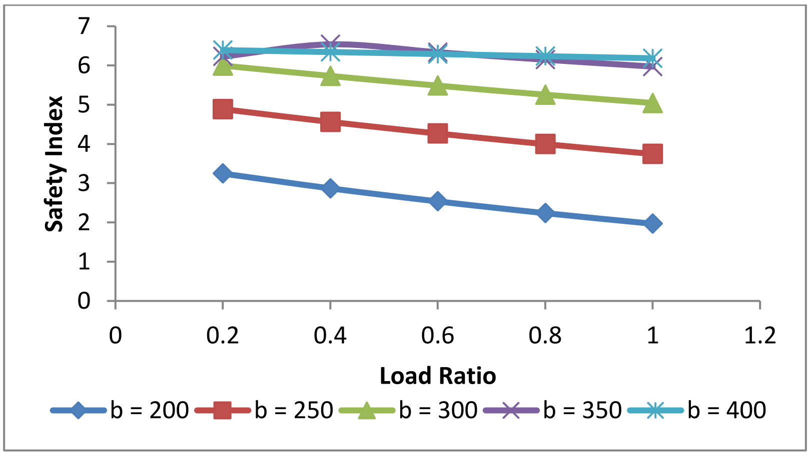

Fig. 13: Safety index against load ratio at varying column depth (bending failure mode)

5. Conclusions

The results of safety analysis of a solid timber column for varying load ratios, radius of gyration, axial load, lateral load, depth and length considering compression, bending and compression-bending failure modes using First Order reliability approximation have been presented. The results of the reliability analysis showed that reliability indices increased with increase in radius of gyration, decreased with increase in axial and lateral load ratio for compression, bending and compression-bending interaction failure modes considered. The reliability index was also found to decrease with increase in length of column considering the three failure modes and using a column length of 6m and 7m will jeopardize the safety of the column. The reliability indices were also found to decrease with increase in axial and lateral loads for varying lengths of column considering compression and compression-bending interaction and bending and compression-interaction failure modes. However, adequate and suitable choice of column dimensions having a lower slenderness ratio will enhance the safety of the column.

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